# INVERTING THE CLARINET

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**ABSTRACT**

Physical-modelling based sound resynthesis is considered by estimating physical model parameters for a clarinet-like system. Having as a starting point the pressure and flow signals in the mouthpiece, a two-stage optimisation routine is employed, in order to estimate a set of physical model parameters that can be used to resynthesise the original sound. Tested on numerically generated signals, the presented inverse-modelling method can almost entirely resynthesise the input sound. For signals measured under real playing conditions, captured by three microphones embedded in the instrument bore, the pressure can be successfully reproduced, while uncertainties in the fluid dynamical behaviour reveal that further model refinement is needed to reproduce the flow in the mouthpiece.

## 1. INTRODUCTION

Resynthesis of realistic sounds using physical models can be achieved by estimating the value of the parameters that govern the instrument oscillations. Even though this is possible by carrying out a system identification process that estimates fitted parameters [1], this study aims to estimate parameters that have a direct physical interpretation; this is a pre-requisite for achieving the long term aim of the authors to perform physically-based transformation by altering these parameters. Since musical instruments behave nonlinearly, a physical model may fail to inherit all the aspects of the real sound-production mechanism [2]. In the clarinet mouthpiece two types of non-linearities manifest themselves. A mechanical one, due to the interaction of the reed with the mouthpiece lay, and one attributed to fluid dynamical effects. The former can in principle be incorporated in a lumped reed model by estimating its parameters using a mechanical description of the system [3], or by using quasi-static measurements [4, 5]. However, under real playing conditions, the system may exhibit dynamic behaviour that cannot be captured by quasi-static analysis. The present study estimates lumped reed model parameters from oscillations generated naturally by an instrumentalist and investigates to what extent this model can follow such measurement signals.

Focusing on the clarinet, this paper presents an inverse modelling procedure that takes as input the pressure and flow signals inside the clarinet mouthpiece and estimates the physical model parameters needed to resynthesise a sound as close as possible to the one performed by the real player. This procedure consists of a two-step routine; two different optimisation methods are used, the first one to bring the problem to a good starting point, and the second to minimise the difference of the real and the estimated signals in the mouthpiece. Rather than using a perceptual criterion for the objective function [2], we intend to study to what extent the model will achieve to reproduce the original waveforms, since with perceptual criteria it is not guaranteed that a physically meaningful set of parameters is extracted.

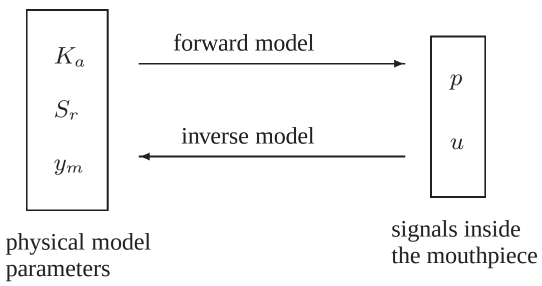


Figure 1: *Forward and inverse modelling of the reed-mouthpiece system, where $K_a$ is the effective stiffness, $S_r$ the effective reed surface, $y_m$ the closing position of the reed and $p$ and $u$ the pressure and flow inside the mouthpiece.*

To simulate a sustained clarinet tone, a non-linear excitation mechanism (reed-mouthpiece-lip system) is coupled to a linear resonator (bore). Using a set of parameters for the physical model, the pressure and flow signals in the mouthpiece can be generated numerically (i.e. a forward model). These signals can also be measured experimentally using wave separation techniques [6, 7]. The process presented in the current paper uses the signals in the mouthpiece as an input to estimate the parameters (i.e. an inverse model), as depicted in Figure 1. Section 2 describes the lumped reed model that interacts with the resonator to generate the oscillations of the instrument and Sections 3 and 4 present the two optimisation methods as applied to numerically generated signals. The experimental setup used to obtain the signals under real playing conditions is described in Section 5, and the application of the whole optimisation routine based on the measured signals is discussed in Section 6. Finally, the results of the process and future objectives are summarised in Sections 7 and 8.

## 2. A LUMPED REED MODEL

The reed oscillation is simulated using a lumped model of the reed-mouthpiece-lip system [3, 8]. In previous studies the mechanical parameters of this model were estimated from a two-dimensional distributed model of a vibrating reed clamped to the mouthpiece [9]. The resulting lumped model follows the mechanical behaviour of the distributed model, taking into account the effect of the players lips, as well as the interaction of the reed with the mouthpiece lay. It can be argued that keeping the effective mass and damping of the reed constant in a lumped model formulation captures most of the dynamics of the system, at least for small amplitude oscillations [10]. Hence, the equation of motion for the lumped reed model is

$$m\frac{d^2y}{dt^2} + g\frac{dy}{dt} + K_a(\Delta p)y = \Delta p, \tag{1}$$

where $y$ is the reed displacement, $m$ the mass per unit area and $g$ the damping per unit area. The effective stiffness per unit area, $K_a$, is treated as a function of $\Delta p$, the pressure difference across the reed, thus rendering the model able to incorporate the quasi-static mechanical non-linear behaviour of the system.

Concerning the flow inside the mouthpiece, an air jet with a varying height is formed in the reed channel, as predicted by boundary layer flow theory. If $\alpha$ is the “vena contracta” factor and $S_f$ the opening surface , it can be assumed that [8]

$$\alpha S_f \approx \lambda h, \tag{2}$$

where $\lambda$ is the effective width of the reed and $h$ the reed opening. The flow ($u$) inside the reed channel is expressed by Bernoulli's equation for ideal fluid flow [11]

$$\frac{1}{2}\rho|u|^2 + p = \text{const.} \tag{3}$$

where $p$ is the pressure in the mouthpiece, and the flow induced by the oscillation of the reed is

$$u_r = \frac{dy}{dt}S_r, \tag{4}$$

with $S_r$ the effective moving surface of the reed.

The mouthpiece pressure $p$ can be decomposed into a wave going into $(p^+)$ and out $(p^-)$ of the bore, which are related to the total volume flow $u = u_r + u_f$ by

$$Z_0 u = p^+ - p^-, \tag{5}$$

where $Z_0$ is the characteristic impedance at the mouthpiece entry. Combining equations (3) and (5) yields the non-linear equation for $u_f$

$$\text{sign}(u_f)\frac{\rho}{2(\lambda h)^2}u_f^2 + Z_0 u_f + (2p^- - p_m + Z_0 u_r) = 0, \tag{6}$$

where $p_m$ is the blowing pressure and $\rho$ the air density.

The above lumped element is coupled to a digital bore model, constructed using wave variables [12], to create a feedback loop that completes the digital representation of the instrument. A theoretical approximation of the parameters of this physical model enables the synthesis of the pressure and flow signals in the mouthpiece. These numerically synthesised signals can be used as an input to the presented optimisation routine, estimating a new set of physical model parameters that can be directly compared to the theoretical ones used during the simulation. Furthermore, using the same physical model to resynthesise these signals gives us the opportunity to directly compare the input signals with the ones obtained with the estimated parameters. Since they have been both numerically generated by the same model, any differences between these signals will have been introduced during the optimisation process.

This lies in contrast to the optimisation based on input signals from naturally performed sounds, since in that case errors can also be attributed to (1) the inability of the model to capture all the physical details of the experiment and (2) measurement errors.

## 3. FIRST OPTIMISATION STEP

The first step towards the parameter optimisation is based on the simplyfying assumption that the the reed displacement $y$ is proportional to the pressure difference $\Delta p$ across it [13]:

$$y = C\Delta p = C(p_m - p), \tag{7}$$

where $C$ is the compliance of the reed [14] and $p$ the pressure inside the mouthpiece. The reed opening $h$ can be related to $y$ as

$$h = y_m - y, \tag{8}$$

with $y_m$ the closing position of the reed.

Under this assumption the effects of inertia forces due to the mass of the reed and frictional forces due to internal damping are neglected. It can be argued that even though these forces might dominate the transient behaviour of the system, their effect almost vanishes at steady state (see Figures 4 and 5 in [8]), and it is the steady state of the input signal that is going to be used for optimisation purposes, thus allowing the above assumption to be made.

The total flow into the mouthpiece as a function of the reed displacement $y$ is

$$\begin{aligned} u &= u_f + u_r \\ &= (-\lambda y + \lambda y_m)\sqrt{\frac{2(p_m - p)}{\rho}} - CS_r\frac{dp}{dt} \\ &= c_1\sqrt{\frac{2}{\rho}}(p_m - p)^{3/2} + c_2\sqrt{\frac{2}{\rho}}(p_m - p)^{1/2} + c_3\frac{dp}{dt}, \end{aligned} \tag{9}$$

with $\begin{cases} c_1 = -C\lambda \\ c_2 = y_m\lambda \\ c_3 = -CS_r \end{cases}$

Since the effective stiffness $K_a$ is the reciprocal of the compliance $C$ we can estimate physical model parameters from the arbitrary parameters $c_1$, $c_2$ and $c_3$ using the following relationships:

$$\begin{cases} K_a = -\lambda/c_1 \\ y_m = c_2/\lambda \\ S_r = \lambda c_3/c_1 \end{cases},$$

Note that during this process $h$ is not restricted to positive values. This can be enforced by adding a constraint of the form $c_2 + c_1\Delta p > 0$, or it can be dealt with during the fine-tuning stage, at the second optimisation step.

The main feature of this technique stems from the fact that including the reed induced flow $u_r$ in equation (9) brings into effect the derivative of the pressure with respect to time ($dp/dt$). Since the reed is assumed to move in phase with the pressure difference across it, the model can now distinguish between the opening and closing phases of the reed motion. As such, the two branches that appear if we plot flow over pressure difference can be treated separately. This branch separation allows an optimisation process to be

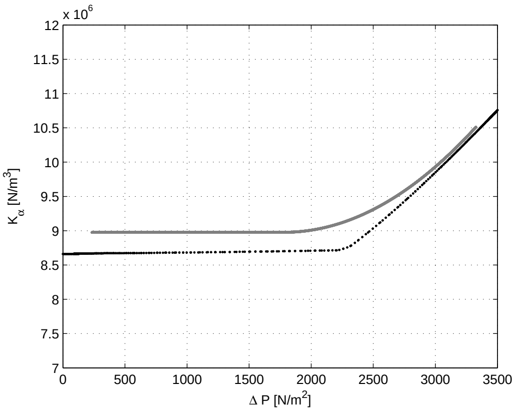


Figure 2: *Effective stiffness per unit area as predicted by the mechanical properties of the reed [9] (dotted-black) and as estimated from the numerically synthesised signals (grey).*

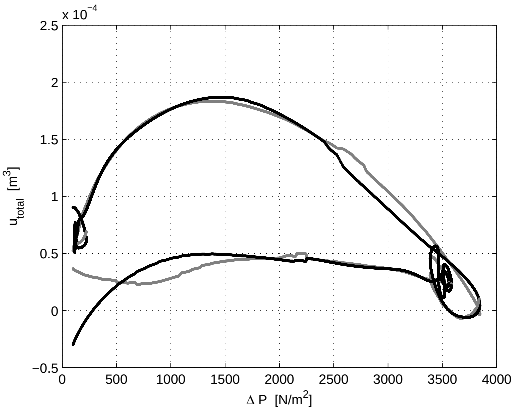


Figure 3: *Flow into the mouthpiece over pressure difference, for the original model (black) and as calculated using the estimated parameters (grey).*

applied twice; once for the opening state of the reed ($dp/dt < 0$) and once for the closing state ($dp/dt > 0$). Taking as our objective function the mean square error between the original flow signal and the estimated flow as calculated from equation (9), and using the Nelder-Mead optimisation algorithm [15, 16], we can get a first estimate for $K_a$, $S_r$, $y_m$ and $p_m$ [13]. These parameters can be then fed into the lumped model to resynthesise the signals in the mouthpiece. In the case of $K_a$, and since it is expected to behave as a function of pressure difference, the estimated value is treated as the (constant) value of $K_a$ for a low pressure difference (when the reed behaves linearly, for there is no interaction with the lay), whereas for higher values of $\Delta p$ it rises to around 1.5 times its value, as predicted by theory (see Figure 2).

One way to evaluate the obtained results of this first estimation of the physical model parameters is to compare the flow signal that was used as an input with the estimated flow that is synthesised using the set of the estimated parameters. These are plotted in Figure 3 over the pressure difference across the reed. It can be deduced that for pressure difference values within the range $[500, 3000]\mathrm{N/m^2}$ the estimation is good enough to be used as a starting point for a second optimisation method. It is also possible to feed the new parameter set to the lumped model and compare the signals in the mouthpiece that have been used as input and the resynthesised signals. A zoomed-in version of these signals is plotted in Figure 4. Finally the values of the estimated parameters and the parameters used to create the input signals are listed in table 1,

Table 1: *Theoretical vs. estimated parameters.*

| | theoretical | estimated (I) | estimated (II) | units |
|---|---|---|---|---|
| $K_a$ | $8.66 \cdot 10^6$ | $8.97 \cdot 10^6$ | $8.67 \cdot 10^6$ | $\mathrm{N/m^3}$ |
| $S_r$ | $7.61 \cdot 10^5$ | $8.42 \cdot 10^5$ | $8.33 \cdot 10^5$ | $\mathrm{m^2}$ |
| $y_m$ | $4 \cdot 10^{-4}$ | $3.2 \cdot 10^{-4}$ | $3.68 \cdot 10^{-4}$ | m |
| $p_m$ | 1800 | 1919 | 1825 | Pa |
| $\lambda$ | 0.013 | — | 0.0142 | m |
| $m$ | 0.05 | — | 0.044 | $\mathrm{Kg/m^2}$ |
| $g$ | 3000 | — | 3805 | 1/sec |

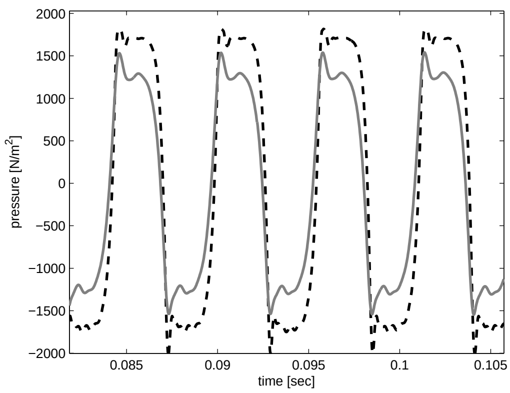


Figure 4: *Pressure signals in the mouthpiece for the original model (dashed-black) and as synthesised using the estimated parameters after the first optimisation step (grey).*

including the parameters estimated from the second optimisation method, as explained in the next section.

## 4. SECOND OPTIMISATION STEP

Having established a method to get a first estimate of the physical model parameters allows us to proceed with a second optimisation method that completes the routine presented in the current paper. The parameters obtained so far enable the synthesis of oscillatory signals; they lie within a range so that the simulation of sustained clarinet notes is possible. As seen in Section 3 it is possible to recreate a note using these parameters, avoiding any regions where blowing thresholds are not reached [17]. It remains to fine-tune the estimated parameters so that we get a better match for the original and the resynthesised signals.

Using as our objective function the mean square error of the pressure signals at the steady state, we employ the Rosenbrock method [18, 19] to locate the optimum set of parameters. The Rosenbrock algorithm is a direct search method, that can go through an $n$-dimensional search space. Starting with a set of $n$ orthogonal directions, the algorithm moves towards those directions that reduce the value of the objective function (for minimisation problems) and then it changes the directions to a new orthogonal set, more likely to yield better results. It has the advantage that by changing the set of the search directions, it can adapt to narrow "valleys" that can appear in the search-space. In addition, by expanding the motion towards successful directions and reducing that towards unsuccessful ones, it has the ability to avoid getting trapped within regions of local minima.

In our application, we run the clarinet simulation after each parameter search within the Rosenbrock algorithm, to synthesise the pressure signal in the mouthpiece and compare it to the original one. In contrast to the first optimisation step, it is now possible to include in the model all the physical parameters that govern the oscillations of the system, namely $K_a, S_r, y_m, p_m$, mass $m$, damping $g$ and effective width $\lambda$. In addition, for $K_a$ a second parameter is introduced, corresponding to its maximum value at high $\Delta p$.

The resynthesised pressure signal should, at every iteration, lie closer to the pressure signal that was used as an input. Thus by starting with two signals that lie reasonably close to each other, something achieved in the previous section, it is possible to reach a suitable set of parameters that produces almost identical results. Again, working only with numerically generated signals, and since we are using the same model to create both the input and the resynthesised signal, an almost perfect match is required to indicate the efficiency of the method. A comparison between the input and the resulting pressure and flow in the mouthpiece can been seen in Figure 5. This two-stage optimisation routine can also be applied to signals measured under real playing conditions, as long as the pressure and flow signals are known.

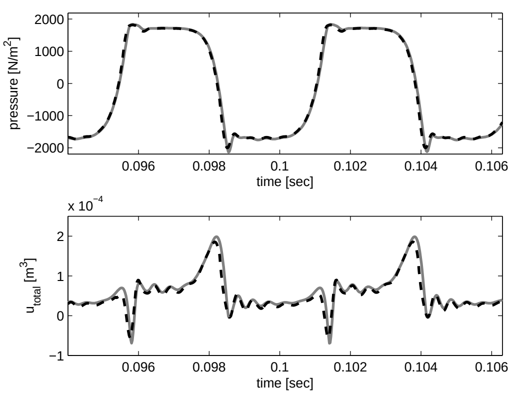


Figure 5: *Pressure signals in the mouthpiece for the original (dashed-black) and the resynthesised (grey) sound (top) and the corresponding flow signals (bottom).*

## 5. SIGNAL MEASUREMENT

The experimental data is obtained from experiments with blowing a simplified clarinet, the schematic bore profile of which is shown in Figure 6. Following [12], the mouthpiece is modelled as a cylindrical plus a conical section, where the first is a heavily simplified axially symmetric representation of the entry of the real mouthpiece; this approach is somewhat justified by the fact that the dimensions of the real mouthpiece geometry are small compared to the wavelength. Another reason to use such a simple model is that the fluid dynamics in this area are generally much more complicated than in the remaining part of the bore, involving complex phenomena such as jet formation and its attachment and de-attachment from the side wall. For dynamic cases (i.e. when the reed moves), this behaviour is not yet understood well [8, 20, 21], and in the light of such modelling uncertainties, the best approach seems to use a simple model. The dimensions of the conical section as well as the step in the radius can be measured directly; the length of the cylindrical section was determined from the measured volume of the section ($\approx$ 3 ml).

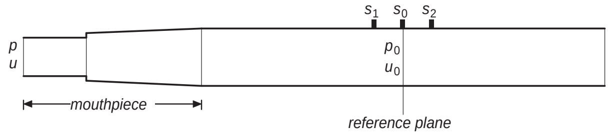


Figure 6: *Schematic depiction of the experimental setup.*

In the experiments, the player generates a sustained note of about 4 seconds. The signals captured by the three microphones embedded in the side wall of the main cylindrical bore are then processed using adaptive delay-loop filtering in order to derive the pressure and flow at the reference plane. This method involves estimation of the parameters that model the transfer function between the microphones, adapting to the playing conditions (the reader is referred to [7] for a more detailed description).

Once the pressure ($p_0$) and volume flow ($u_0$) at the reference plane are measured, classical transmission-line theory using ABCD matrices [22] is applied in order to derive the corresponding pressure ($p$) and flow ($u$) at the mouthpiece entry. Zero-phase lowpass filtering with a 7.25 kHz cut-off is applied to both signals in order to remove high frequency errors that arise from the singularities inherent to the three-microphone adaptive delay-loop filtering method.

## 6. OPTIMISATION ROUTINE FOR THE MEASURED SIGNALS

Having obtained the signals of pressure and flow in the mouthpiece we can directly proceed to the first step of our optimisation routine. Equation (9) is used to form the objective function for the Nelder-Mead method. Working on a "slice" of the measured signal that closely resembles the steady state of a sustained note, it is still possible to distinguish between the opening and closing states of the reed motion, by calculating $dp/dt$ from the pressure signal. As in Section 3, optimising separately for each branch and averaging the obtained results gives a first estimate for the physical model parameters, the validity of which is suggested by the comparison of the measured flow and the calculated flow, as depicted in Figure 7.

Since $K_a$ is known not to be constant, it is possible to get a better estimation by feeding the rest of the estimated parameters to equation (9) and solving for $K_a$ as a function of $\Delta p$ (since $u$ is known from the measurements). The pressure and flow signals in the mouthpiece, as resynthesised using the parameters estimated form this first-step optimisation, are compare to the original ones in Figure 8. The main cause of the deviation of the flow signal of

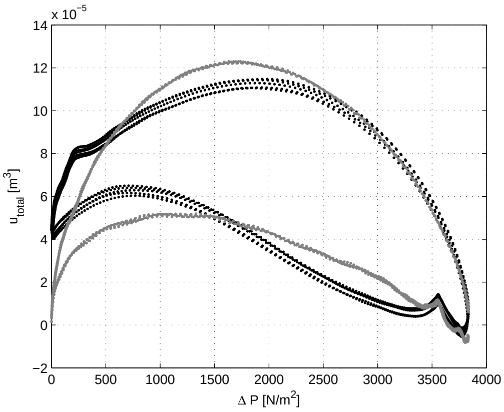


Figure 7: *Flow into the mouthpiece over pressure difference, for the measured data (dotted-black) and as calculated using the estimated parameters after the first optimisation step (grey).*

this figure, as compared to the one in Figure 7, is the inclusion of arbitrary mass and damping parameters during its resynthesis.

In order to transfer the above results to the second stage of our optimisation routine, we have to adapt our model to the dimensions of the experimental setup, used to obtain the measurements, as described in Section 5. The bore impedance ($Z_0$) as seen from the reference plane was calculated from theory [22]. A frequency-domain comparison between $P_0(\omega)$ and $Z_0(\omega)U_0(\omega)$ showed a good match between theory and measurement, validating the derivation of a suitable reflection function from $Z_0(\omega)$ for use in the time-domain clarinet model.

Another problem that has to be tackled before proceeding with the Rosenbrock algorithm is that the measured pressure signal and the resynthesised signal from the lumped model might not be in phase. This was not a problem in Section 4, as the two signals were generated using the same model. Now it is not guaranteed, constituting the synchronisation of the two signals necessary. This can be achieved by shifting the numerically synthesised signal for the required amount of samples, until it lies in phase with the original (measured) signal. (Note that such a synchronisation will be repeated several times during the optimisation routine, to ensure that the compared signals lie in phase.) The results of the whole optimisation routine, applied to a small, steady part of the measured signal, are shown in Figure 9.

## 7. DISCUSSION

The pressure signal in the mouthpiece can be resynthesised to match the measured one. For the flow, however, even though the resynthesised signal lies closer to the original one after the second optimisation stage, a perfect match was not obtained. The use of a different objective function for the Rosenbrock algorithm, that included both the pressure and flow signals in the mouthpiece, did not improve the estimation process. This indicates that the focus should be shifted towards improving the model rather than the optimisation routine.

For the measured signals there are non-linearities and uncertainties in the fluid dynamics that are not incorporated in the physical model. These may stem from (1) a yet unpredictable behaviour of the "vena contracta" factor at low $\Delta p$ regimes [23], and (2) the effects of turbulence. On the other hand, applying the optimisation routine on numerically generated data succeeded in resynthesising both pressure and flow signals. Since in that case both the input and estimated signals are generated using the same model, the relation between pressure and flow in the mouthpiece remains the same.

Future studies will focus on attributing the difference of the flow signals to appropriate physical phenomena, so that the model can regenerate both signals in the mouthpiece. It has to be pointed out here that the goal of our study remains to estimate parameters that have a physical meaning. Thus, even though black-box techniques could parameterise the reed non-linearity, most of the resulting parameters would not have a direct physical interpretation. Hence such an approach would be less in line with our objectives.

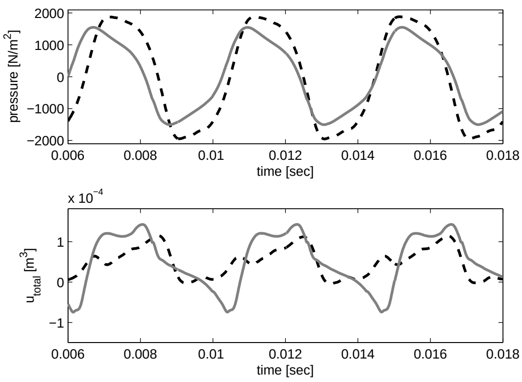


Figure 8: *Pressure signals in the mouthpiece for the original (dashed-black) and the resynthesised (grey) sound (top), and corresponding flow signals (bottom), after the first optimisation step.*

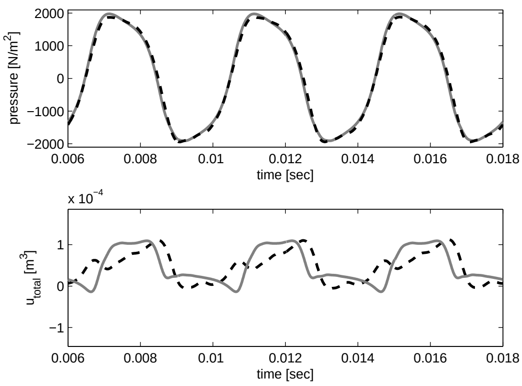


Figure 9: *Pressure signals in the mouthpiece for the original (dashed-black) and the resynthesised (grey) sound (top), and flow signals (bottom), after the second optimisation step.*

## 8. CONCLUSIONS

A two-stage optimisation routine, that uses the Nelder-Mead and Rosenbrock algorithms, can estimate physical model parameters, suitable for clarinet sound resynthesis. Starting from signals measured under real playing conditions, the pressure signal can be regenerated using the estimated parameters as input to the model. For the flow in the mouthpiece, though, our model has to be refined, in order to improve the resynthesis accuracy.

## 9. ACKNOWLEDGEMENTS

The authors would like to thank Giovanni de Sanctis (Queen's University Belfast), for providing the apparatus to measure the internal bore pressure signals.

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