MODELING OF NONLINEAR AUDIO SYSTEMS USING SWEPT-SINE SIGNALS: APPLICATION TO AUDIO EFFECTS

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ABSTRACT

In this paper a new method for analysis and modeling of nonlinear audio systems is presented. The method is based on swept-sine excitation signal and nonlinear convolution firstly presented in [1, 2]. It can be used in nonlinear processing for audio applications, to simulate analog nonlinear effects (distortion effects, limiters) in digital domain.

1. INTRODUCTION

As far as audio digitalization is concerned, the analog audio recordings are converted into digital files, the analog tapes are replaced with digital medias and the audio devices are price-out and replaced by the digital ones. Even though it is claimed that analog audio still offers the best sound quality, we cannot imagine today life without digital signal processing.

Moreover, rapid development of computer industry results in digitalization of analog audio effects. Nevertheless, several analog audio processing devices exhibit nonlinearities, which are not easy to simulate. Volterra series is an approach known for several years, but is not suitable for strong harmonic distortion [3]. Several other nonlinear models have been proposed, such as neural network model [4], MISO model [5], NARMAX model [6, 7], hybrid genetic algorithm [8], extended Kalman filtering [9], particle filtering [10].

In this paper, we propose a new method for identifying nonlinear systems, based on an input exponential swept-sine signal [1, 2], allowing a robust and fast one-path analysis and model estimation of the unknown nonlinear system under test. The analysis method is discussed in section 2. The nonlinear model used for the synthesis of distorted signal consists of several parallel branches, each branch consisting of a nonlinear function and a linear filter. The nonlinear functions can be chosen either based on partial knowledge of the nonlinear system or, based on any mathematical series such as power series. The model is discussed in section 3. To show the efficiency of the method an audio limiter (the case of a strong harmonic distortion) is analyzed, its nonlinear model is identified and tested on both musical and speech signals.

2. ANALYSIS OF NONLINEAR SYSTEMS

The analysis method is partly based on nonlinear convolution method presented in [1, 2]. The method uses a swept-sine signal (also called a chirp), exhibiting an exponential instantaneous frequency, as excitation signal and allows the characterization of a nonlinear system (NLS) in terms of harmonic distortion at several orders.

The block diagram of the method is shown in Fig. 1. First, an exponential swept-sine signal $x_s(t)$ is generated and used as the input signal of the nonlinear system under test. The excitation swept-sine signal $x_s(t)$ is defined as

$$x_s(t) = \sin\left\{2\pi L\left[\exp\left(\frac{f_1 t}{L}\right) - 1\right]\right\}, \tag{1}$$

where

$$L = \text{Round}\left(\frac{\hat{T} f_1}{\ln\left(\frac{f_2}{f_1}\right)}\right), \tag{2}$$

where f_1 and f_2 are start and stop frequencies and $\hat{T}$ is the time duration of the swept-sine signal. The rounding operator in Eq.(2) is due to the condition of synchronized higher orders as depicted in Fig.2.

The distorted output signal $y_s(t)$ of the nonlinear system is recorded for being used for the so-called nonlinear convolution [1]. Next, the signal noted $\tilde{x}_s(t)$ is derived from the input signal $x_s(t)$ as its time-reversed replica with amplitude modulation such that the convolution between $x_s(t)$ and $\tilde{x}_s(t)$ gives a Dirac delta function $\delta(t)$. The signal $\tilde{x}_s(t)$ is called "inverse filter" [1].

Then, the convolution between the output signal $y_s(t)$ and the inverse filter $\tilde{x}_s(t)$ is performed. The result of this convolution can be expressed as

$$y_s(t) * \tilde{x}_s(t) = \sum_{i=1}^{\infty} h_i(t + \Delta t_i), \tag{3}$$

where $h_i(t)$ are higher-order impulse responses and Δt_i are the time lags between the first and the i-th impulse response. Since

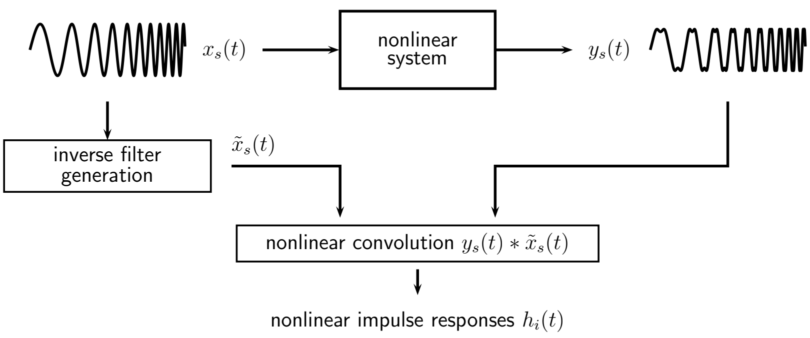

Figure 1: *Block diagram of the nonlinear convolution process in NLS identification.*

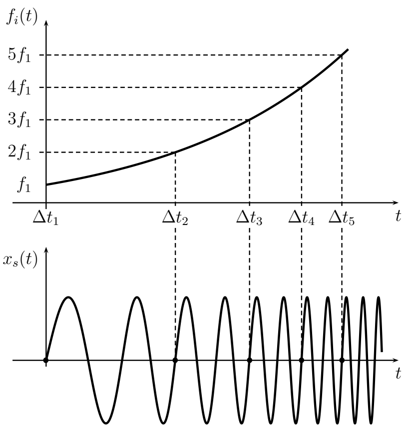

Figure 2: *Swept-sine signal $x_s(t)$ in time domain (below), with the time length chosen according to instantaneous frequency $f_i(t)$ (above).*

the nonlinear impulse response consists of a set of higher-order impulse responses that are time shifted, each partial impulse response can be separated from each other, as illustrated by Fig. 3.

The set of higher-order nonlinear impulse responses $h_i(t)$ can be also expressed in the frequency domain. The frequency response functions of higher-order nonlinear impulse responses $h_i(t)$ is then their Fourier transforms

$$H_i(f) = \mathbf{FT}\left[h_i(t)\right]. \tag{4}$$

The frequency responses $H_i(f)$ represent the frequency dependency of higher-order components. The frequency response $H_1(f)$ is consequently the response corresponding to the linear part of the system. Similarly, the frequency response $H_i(f)$ ($i > 1$) may be regarded as the system frequency response, when considering only the effect of the input frequency f on the i-th harmonic frequency if of the output.

3. MODEL IDENTIFICATION

The frequency responses $H_i(f)$ are next used for the identification of the nonlinear model of the system under test. The model is

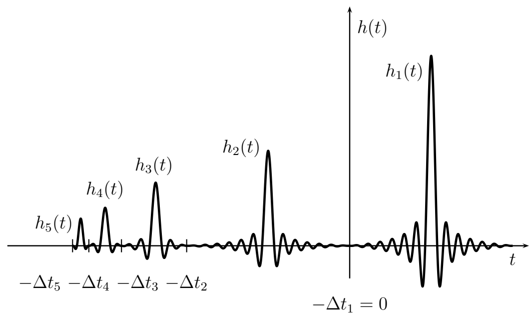

Figure 3: *Result of the nonlinear convolution process $y_s(t) * \tilde{x}_s(t)$ in the form of set of higher-order nonlinear impulse responses $h_i(t)$.*

shown in Fig. 4. It is made up of N parallel branches, each branch consisting of a linear filter $A_n(f)$. The input signals $g_n[x(t)]$ are known linear and/or nonlinear functions of $x(t)$. This model is similar to the Multiple Input Single Output model proposed in [5].

The linear filters $A_n(f)$ of the nonlinear model can be moreover derived in the time domain as impulse responses $a_n(t)$ related to the n-th branch of the MISO-based nonlinear model. The output

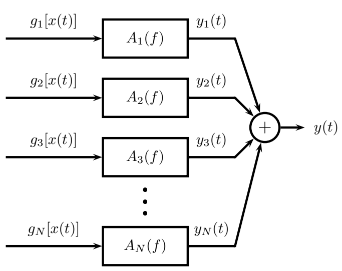

Figure 4: *Nonlinear model with input signals $g_n[x(t)]$ and linear filters $A_n(f)$, $n \in [1, N]$.*

signal $y(t)$ of the nonlinear system can then be expressed as

$$y(t) = \sum_{n=1}^{N} \int_{-\infty}^{\infty} g_n[x(\tau)] a_n(t-\tau) d\tau, \quad (5)$$

where N is the number of input signals of the MISO-based nonlinear model.

Frequency response functions $H_i(f)$ being experimentally obtained and the nonlinear MISO model inputs $g_n[x(t)]$ being chosen, the identification consists in the resolution of a linear system of N equations using the least squares method. First, the coefficients $c_{n,k}$ of Discrete Fourier Series of the functions $g_n[x(t)]$ are calculated as

$$c_{n,k} = \frac{2}{M} \sum_{m=0}^{M-1} g_n \left[\sin(\frac{2\pi}{M} m) \right] \exp\left(-j \frac{2\pi}{M} km \right), \quad (6)$$

for an input signal being a discrete-time harmonic signal of length M. Next, the following set of linear equations with unknown $A_n(f)$ is solved

$$H_i(f) = \sum_{n=1}^{N} A_n(f) c_{n,i} + \mathrm{Res}(f), \quad (7)$$

for $i \in (1, I)$ and $n \in (1, N)$, $\mathrm{Res}(f)$ being the residue. As $I \geq N$, there can be more equations than unknowns. To solve the set of equations (7) for $I > N$, the least squares algorithm [11] is applied, minimizing the residue $\mathrm{Res}(f)$.

If the functions $g_n[x(t)]$ are improperly chosen and/or if at least one of the input signals is missing, the value of the residue increases drastically, which makes $\mathrm{Res}(f)$ an *a posteriori* criterium for the choice of input signals $g_n[x(t)]$.

If one of the nonlinear functions $g_n[x(t)]$ produces high harmonic distortion components, the nonlinear aliasing [12] can appear. This can be avoid by choice of the nonlinear functions $g_n[x(t)]$ according to any mathematical series. The most used series is the one based on power series, such as

$$g_n[x(t)] = x^n(t). \quad (8)$$

In such case the nonlinear aliasing can be controlled by the frequency range. The highest frequency must not exceed $f_s/(2N)$, where f_s is the sampling frequency and N is the highest power function in the model. The lowest frequency limit is given as well by the highest power function N. The filters $A_n(f)$ are valid only in frequency band $[Nf_1, f_2]$. For that reason the model should be preceded by a bandpass filter as shown in Fig.5. The amplitude limitation is as well given by the excitation signal $x_s(t)$ used for the analysis. As the nonlinear system was not excited with level higher than the amplitude A of the excitation signal $x_s(t)$ the nonlinear model can be used for signals not exceeding this amplitude level.

4. EXPERIMENTAL MEASUREMENT AND SYNTHESIS

In this section, a real-world NLS is selected to show the effectiveness, accuracy and potential of the proposed method. The system under test is the limiter part of *dbx 266XL* Compressor, Limiter, Gate [13]. The limiter is a well known NLS producing highly distorted output waveforms. The clipping level of the limiter is set to 0.25 V.

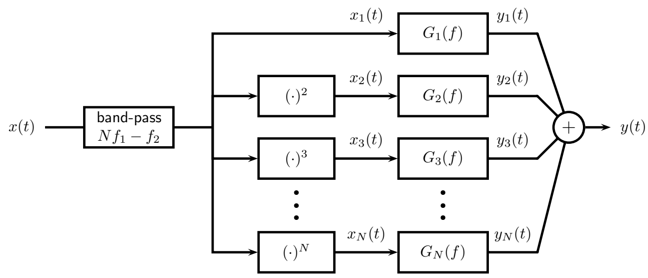

Figure 5: *Generalized Polynomial Hammerstein model (power series nonlinear model).*

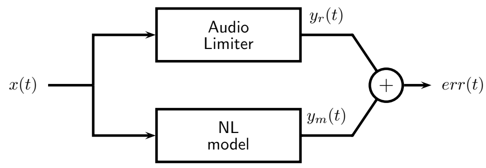

Figure 6: *Block schema of the nonlinear model validation.*

The following process consists in two steps: (a) analysis of the system under test including the nonlinear model identification and (b) comparison of the output signals of the model and of the system under test when excited with a musical and speech signal.

First, the systems response to the excitation swept-sine signal $x_s(t)$, defined in section 2, is convolved with the inverse filter in order to obtain the nonlinear impulse responses. Next, the linear filters of the nonlinear model are estimated using Eq.(6). The functions $g_n(t)$ are chose according the power series (Eq.(8)). Then, once the nonlinear model of the system under test is estimated, the model can be used as a digital nonlinear effect with single input and single output. As explained at the end of the section 3 since the model was acquired using the excitation swept-sine signal with given f_1, f_2 and A, the model can be successfully applied only for input signals with frequency range $Nf_1 : f_2$ and not exceeding the amplitude A.

The measurement conditions are selected as follows: The sampling frequency used for the experiment is $f_s = 96$ kHz. The limiting threshold of the audio limiter is set-up to 0.25 V. The excitation signal is sweeping from $f_1 = 5$ Hz to $f_2 = 6.5$ kHz with amplitude $A = 1$ V. The nonlinear model consists of $N = 7$ branches.

To validate the accuracy of the model the following test is then performed. An input signal is put to the inputs of both, real limiter and its estimated model and the responses are compared. The block schema is depicted in Fig.6. The signal $err(t)$ being the difference between original output $y_r(t)$ and the model output $y_m(t)$ is chosen as a criterion for the comparison.

Two kinds of audio signals has been chosen for the test - a musical signal (sample of piano concerto) and a speech signal (sample of czech poem recital), both with duration of 2 seconds. The results are shown in Figs. 7-10. Each figure has two parts, above - with the input $x(t)$ (green), real-output $y_r(t)$ (blue) and model-output $y_m(t)$ (red) and below - with the residual error $err(t)$. In all four figures the real-output and model-output match rather precisely.

5. CONCLUSIONS

In this paper an application of the method for analysis and modeling of nonlinear systems (NLS) using swept-sine signals has been presented. The application of the method has been focused to audio applications and nonlinear effects. The model of nonlinear system (i.e. existing analog effects system producing a nonlinear distortion) can be identified and further used as a replica of the analog effect in digital domain. The method is based on the nonlinear convolution method, with swept-sine input signal, and allows to identify the NLS in a one-path measurement.

6. ACKNOWLEDGMENTS

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7. REFERENCES

[1] E. Armelloni and others, “Non-Linear Convolution: A New Approach for the Auralization of Distorting Systems,” in *AES 110th convention*, Amsterdam, may 2001.

[2] A. Farina, “Simultaneous measurement of impulse response and distortion with a swept-sine technique,” in *AES 108th convention*, Paris, Feb. 2000.

[3] U. Zölzer, *DAFX:Digital Audio Effects*, John Wiley & Sons, 2002.

[4] O. Nelles, *Nonlinear System Identification: From Classical Approaches to Neural Networks and Fuzzy Models*, Springer, Berlin, 2001.

[5] J.S. Bendat, *Nonlinear System Techniques and Applications*, John Wiley & Sons, New York, USA, 1998.

[6] F. Thouverez and L. Jezequel, “Identification of NARMAX models on a modal base,” *J. Sound. Vib.*, vol. 189, no. 2, pp. 193–213, 1996.

[7] H. E. Liao and W. S. Chen, “Determination of nonlinear delay elements within NARMA models using dispersion functions,” *IEEE Trans. Instrum. Meas.*, vol. 46, no. 4, pp. 868–872, 1997.

[8] Y. W. Chen et al., “Blind nonlinear system identification based on a constrained hybrid genetic algorithm,” *IEEE Trans. Instrum. Meas.*, vol. 52, no. 3, pp. 898–902, 2003.

[9] H.W. Sorenson, *Kalman Filtering: Theory and Application*, IEEE Press, Montvale, NJ, 1985.

[10] O. Cappé et al., “An overview of existing methods and recent advances in sequential monte carlo,” *IEEE Proc. Signal Proc.*, vol. 95, no. 5, pp. 899–924, 2007.

[11] G. Strang, *Introduction to Applied Mathematics*, Wellesley-Cambridge, Wellesley, Massachusets, 1986.

[12] Y. M. Zhu, “Generalized sampling theorem,” *IEEE Trans. Circuits Syst. II*, vol. 39, pp. 587–588, 1992.

[13] “Dbx : Profesional products, dbx 266xl compressor-gate,” WWW page, 2007.

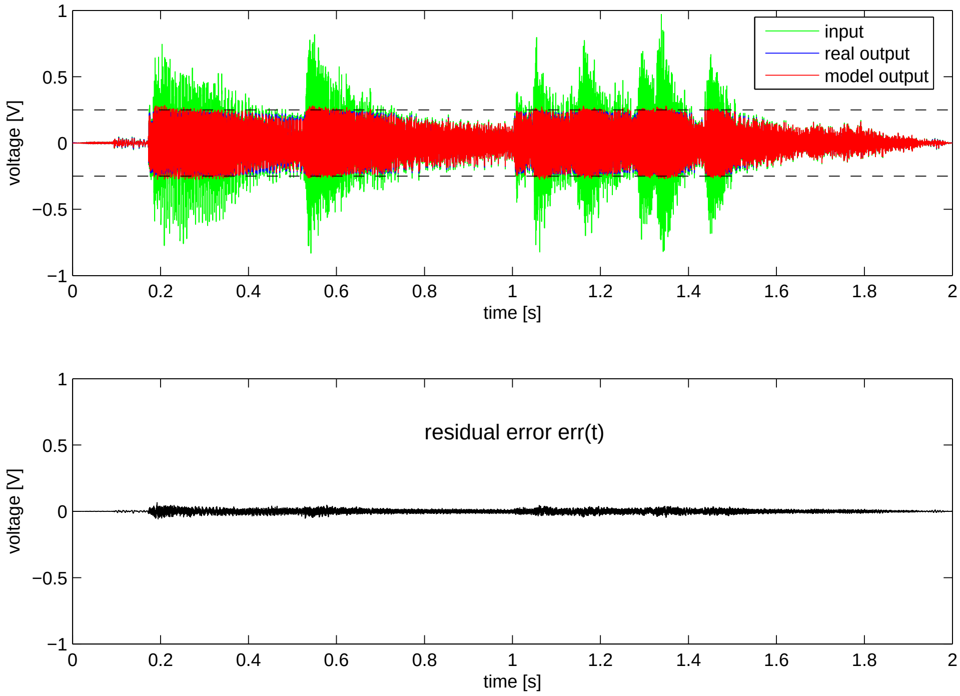

Figure 7: *Music audio file: comparison between real-output and model-output (above) and their difference - residual error (below).*

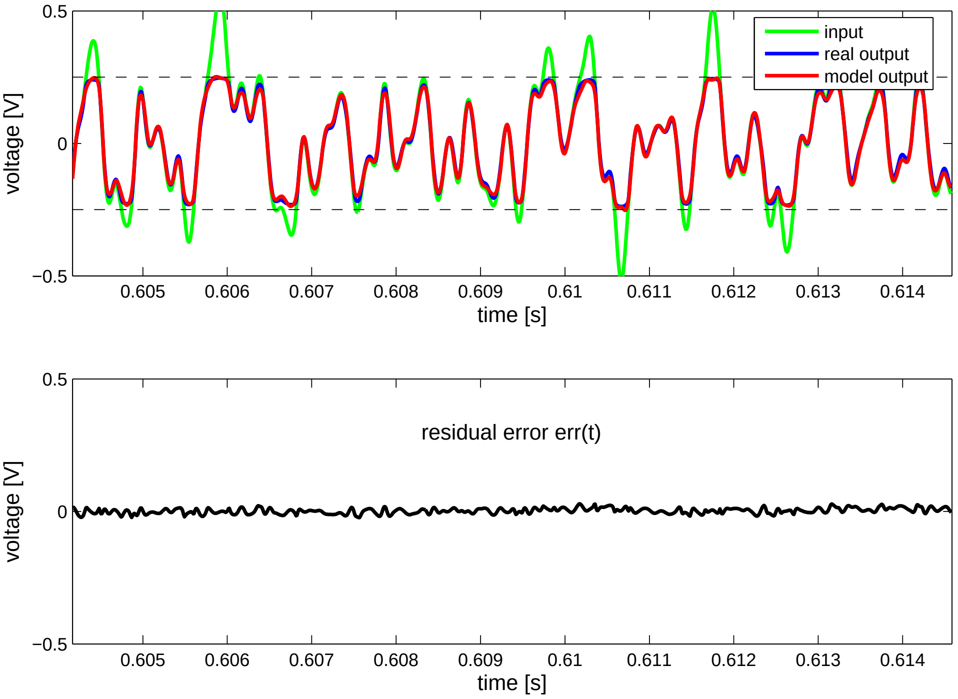

Figure 8: *Music audio file: comparison between real-output and model-output (above) and their difference - residual error (below).*

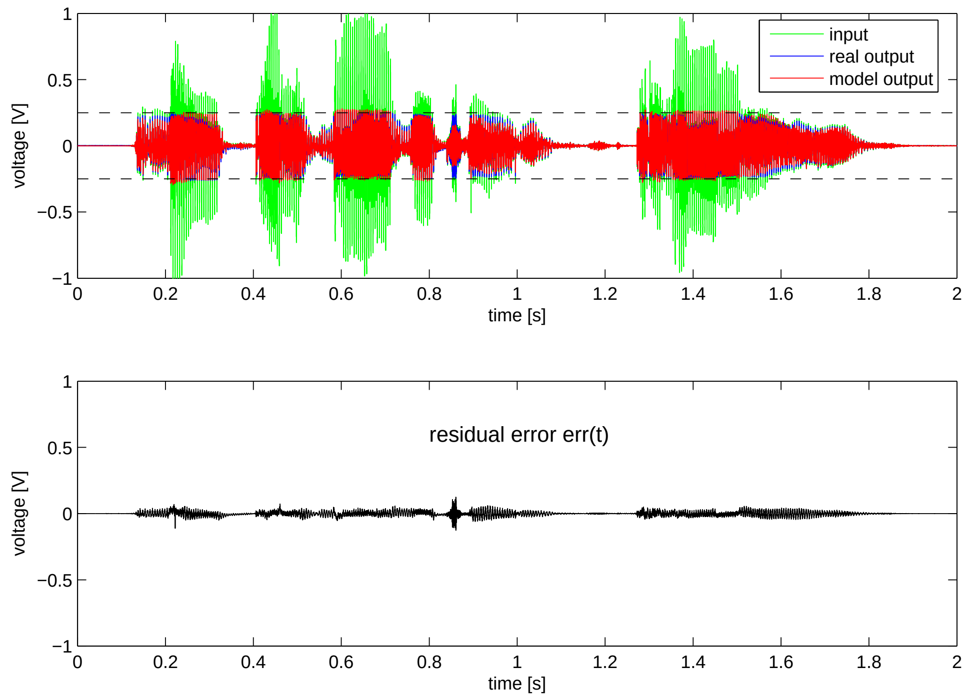

Figure 9: *Speech audio file: comparison between real-output and model-output (above) and their difference - residual error (below).*

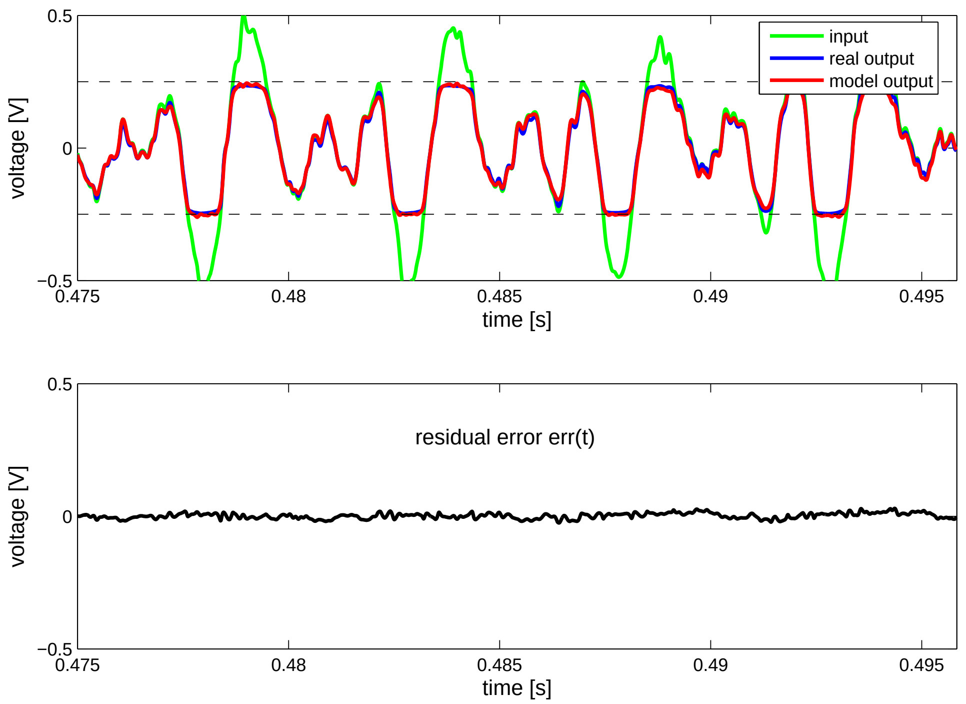

Figure 10: *Speech audio file: comparison between real-output and model-output (above) and their difference - residual error (below).*