ON THE DYNAMICS OF THE HARPSICHORD AND ITS SYNTHESIS

Henri Penttinen

Laboratory of Acoustics and Audio Signal Processing
Helsinki University of Technology
Espoo, Finland
Henri.Penttinen@tkk.fi

ABSTRACT

It is common knowledge that the piano was developed to produce a keyboard instrument with a larger dynamic range and higher sound radiation level than the harpsichord possesses. Also, the harpsichord is a plucked string instrument with a very controlled mechanism to excite the string. For these reasons it is often falsely understood that the harpsichord does not exhibit any dynamic variation. On the contrary, the signal analysis and the listening test made in the this study show that minor but audible differences in the dynamic levels exist. The signal analysis portrays that stronger playing forces produce higher levels in harmonics. The energy given by the player is not only distributed to the plucking mechanism but also carried on from the key to the body. This is evident from the increased level of body mode radiation. A synthesis model for approximating the dynamic behavior of the harpsichord is also proposed. It contains gain and timbre control, and a parallel filter structure to simulate the soundboard knock characteristic for high key velocity tones.

1. INTRODUCTION

The objective of this paper is two fold. Firstly the goal is to clarify the issue of harpsichord dynamics through signal analysis and a listening test. Secondly, the aim is to extend an existing model-based harpsichord sound synthesis algorithm [1] to include realistic modeling of dynamics.

The issue of harpsichord dynamics can easily be considered as an uninteresting topic and solved case, mainly since the piano was developed to obtain some dynamic expression. On the contrary, the theme has still some open issues and there are basically two camps with opposite opinions. One camp is says that the mechanism of the instrument does not allow dynamic variation: "In a harpsichord the energy input is fixed by the nature of the plucking mechanism. The loudness of the note is therefore determined by the efficiency with which string energy is transferred to the soundboard," states [2]. According to [3] "The transient is very reproducable and allows no dynamic variation." Similar remarks can be found in, for example [4, 5]. The other half says that "the tone color and (to some extent) the loudness are both altered when a key is struck more or less hard" [6]. Hall [7] agrees with Benade [6], based on brief informal tests with an FFT (Fast Fourier Transform) analyzer and simulations of the interaction between the plectrum and string. Furthermore, Hall's [7] simulations predict that higher plectrum speeds produce a brighter spectrum. There are also some who are in between, but do not give any specific explanation [8].

The revival of the harpsichord has not only been musical, but also its acoustics has been investigated during the past few decades. General descriptions of the instrument can be found in [4] and [8], and in [2] Fletcher discusses interesting design issues. The behavior of soundboards and air and structural modes have been discussed [9, 10, 11]. Weyer discusses temporal characteristics in the attack [5, 12]. Details of the attack, called pretransients, are discussed in [3], and sonological investigations are presented in [13].

This study tackles the issue firstly from an analysis point of view, by looking at measurement data, and secondly by creating a sound synthesis model for the process. Sound examples related to the work can be found at http://www.acoustics.hut.fi/publications/papers/dafx2006-harpsichord-dynamics/. The remaining parts of this paper are organized as follows. In Sec. 2 the construction of the harpsichord is discussed. Section 3 presents the signal analysis made and Sec. 4 proposes a synthesis model based on this. Section 5 discusses the conducted listening tests and its results. Sections 6 and 7 give a short discussion and conclusions, respectively.

2. HARPSICHORD CONSTRUCTION

The harpsichord is a keyboard instrument belonging to the family of plucked string instruments due to its string excitation mechanism. The harpsichord is played so that a set of keys that form a manual are pushed down and the machinery of the instrument causes a plectrum, also called a quill, to pluck the string. When the key is released, a spring mechanism prevents a repluck to occur. Moreover, a damper comes in contact with the string and dampens it. In contrast to the guitar, where the plucking event can be controlled very freely in many dimensions (force, plucking point, plucking direction, etc.), the excitation process in the harpsichord is bounded by the machinery executing it.

In a harpsichord two to four sets of strings are controlled with one or two manuals. The string sets are also called registers or string choirs, and two of the registers are usually tuned in unison and they are called the $8'$ (eight foot) registers. These two registers differ in the plucking point and are called the $8'$ back and $8'$ front registers, respectively. The plucking point in the $8'$ back register is further away from the player. Often, a third register is also included and it is tuned an octave higher. This register is called the $4'$ register.

One end of the strings is attached to the nut close to the tuning pins. This is the end were the keys are. The other end is attached to a long curved bridge. The string continues after the bridge to a hitch pin, which is on top of the soundboard. The nut is set on a very rigid wrest plank, and the bridge is on top of the soundboard. The soundboard is very thin, about 2 to 4 mm, supported by several ribs. The main function of the soundboard is to amplify the weak sound of the vibrating string. Sometimes, a rose opening is included in the soundboard. Consequently, a Helmholtz resonance is produced with a frequency usually below 100 Hz [2].

The harpsichord used in this study was built in 2000 by Jonte Knif and Arno Pelto. It has characteristics adapted from harpsichords built in Italy and Southern Germany. Moreover, it has three sets of strings, the typical 8′ back and front registers and a 4′ register. Additionally, the highest octave of the 4' register does not have dampers, causing the instrument to have a reverberant response. The registers are controlled through two manuals. The instrument was tuned to the Vallotti tuning [14] to an A_4 that has a fundamental frequency of 415 Hz. Old instruments from the baroque era are typically tuned lower than the current standard, which is 440 Hz or higher.

3. SIGNAL ANALYSIS

3.1. Recorded material

A sound database for a synthesis model [1] was recorded in the large anechoic chamber at Helsinki University of Technology. The recording was done with two pairs of studio microphones placed about 1 m above the soundboard. The data of importance to the dynamics is based on tones played with the 8′ back register and all the C keys (C_2, C_3, C_4, C_5, and C_6), each with three different striking forces or velocities. For practical reasons, these three playing levels are referred to as piano pianissimo (*pp*), mezzo forte (*mf*) and forte fortissimo (*ff*). These dynamic levels were played successively without changing the setup and, therefore, comparing them is legitimate. The fundamental frequencies, f_0, are as follows: C_2 = 62.9 Hz, C_3 = 124.4 Hz, C_4 = 247.6 Hz, C_5 = 495.6 Hz, and C_6 = 991.2 Hz.

3.2. Comparison of partial amplitudes

Next, we take a look at the amplitudes of harmonic partials and how they behave in relation to each other.

3.2.1. Absolute levels

One way to go about investigating the existence of harpsichord dynamics is to analyze the levels of the harmonics of the string vibrations. If only the spectra of string vibrations were compared, the differences in the time domain would remain hidden. Thus, the evolution of string harmonics is observed as a function of time.

Figure 1 illustrates the envelopes of harmonics one (solid line), two (dashed), and three (dotted) for the C_4 tone played with dynamic levels *ff* and *pp*. The first two envelopes of the *ff* tone appear at a higher level throughout, whereas the behavior of the third harmonic seems practically identical. At t = 0.4 s, the differences between the levels for the first, second, and third harmonic are 5 dB, 3 dB, and 0.5 dB, respectively.

Similarly Fig. 2 displays the envelopes of harmonics one (solid line), two (dashed), and three (dotted) for the C_6 tone played with dynamic levels *ff* and *pp*. At t = 0.4 s, the differences between the levels for the first, second, and third harmonic are 0 dB, 2 dB, and 1 dB, respectively. The differences in the levels are smaller for C_6 than for C_4.

The forms of the displayed envelopes are, again, strikingly similar. However, some differences can be noted, for example in the amplitude modulation depth of the envelope of the second harmonic (Fig. 2) is around 0.4 s or for the third harmonic after 1 s. Also, no significant differences or deviating trends in decay times were noted for tones played with different dynamic levels. In addition, it mainly seems that the form of the envelope for a string harmonic is quite independent of the striking velocity. More suggestions for the causes behind the differences are given in the discussion section.

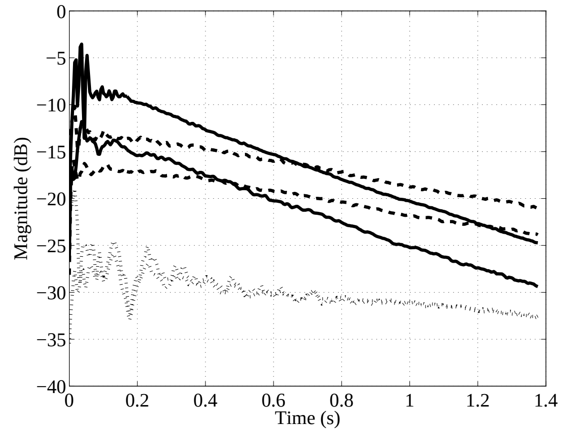

Figure 1: *Envelopes of the first three harmonics for key C_4 tones played as forte fortissimo (ff) and piano pianissimo (pp). Harmonic number and respective line type: 1 - solid, 2 - dashed, 3 - dotted.*

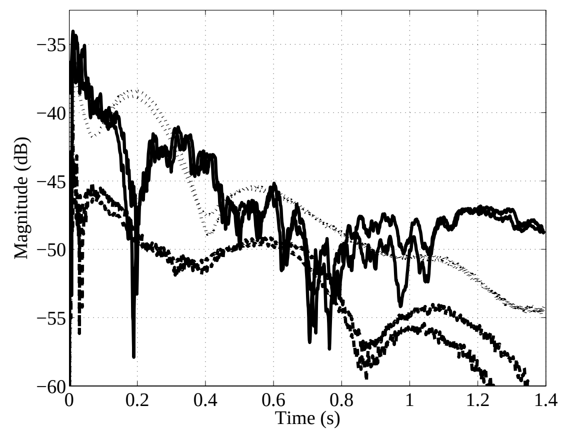

Figure 2: *Envelopes of the first three harmonics for key C_6 tones played as forte fortissimo (ff) and piano pianissimo (pp). Harmonic number and respective line type: 1 - solid, 2 - dashed, 3 - dotted.*

3.2.2. Relative levels of harmonics

Now we compare the levels of harmonics as a function of frequency. As above, the difference in harmonic levels are compared between *pp* and *ff* tones. The *pp* and *ff* tones have been scaled according to the harmonic with the largest level typically being the lowest harmonic. Figure 3 shows the relative difference in levels as a function of harmonic index for tones (a) C_3, (b) C_4, and (c) C_5. There exists a clear increase in level in total, but a relative level difference in harmonics is not evident. In one way, Fig. 3 gives an an estimate of how much the higher harmonics of a *pp* tone should be amplified to simulate the characteristics of a *ff* tone. An increase in the relative level of harmonics as a function of excitation force

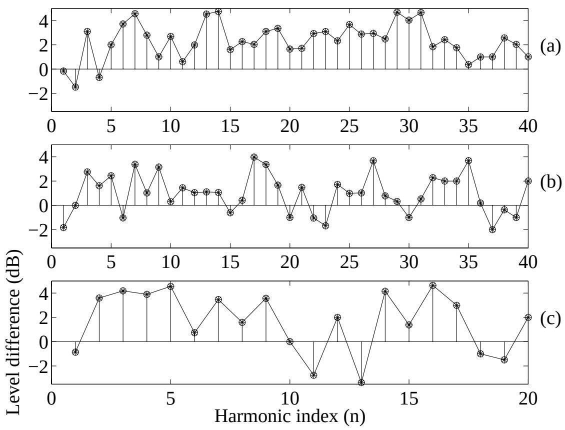

Figure 3: *Relative difference between the level of harmonic envelopes measured at 0.4 s for pp and ff played tones for n harmonics: (a) C_3, (b) C_4, and (c) C_5.*

is typical for musical instruments. However, due to the excitation mechanism this does not seem to apply for the harpsichord, at least to the one measured in this study.

3.3. Comparison of body mode levels

Next, we examine how the soundboard and body modes behave for tones played with different dynamic levels. Now, all recorded levels are compared. Figure 4 shows the magnitude responses of tone C_6 played at *ff* (solid line), *mf* (dashed), *pp* (dotted) dynamic levels. Due to the high f_0 (991.2 Hz) the lowest body modes are nicely separated from the string vibrations and clearly visible. Figure 4 indicates at least a general 10 dB level difference for the lowest body modes ($<$ 300 Hz) from one dynamic level to another. The *ff* tone shows several distinct body resonances, whereas the *pp* tone perhaps only has an emphasis at 50 Hz and the rest of the response is very noise-like [1]. The general level of body modes of the *mf* tone falls between the *pp* and *ff* tones. Similarly as the *ff* tone, the *mf* tone has distinct resonances, but not as prominent.

Figure 5 depicts the magnitude responses of tone C_3 played at *ff* (solid line), *mf* (dashed), *pp* (dotted) dynamic levels. Now the f_0 (124.4 Hz) is considerably lower, but body modes can still be analyzed. The body modes below the fundamental of the string behave for different dynamic levels similarly as for the C_6 case. Body modes can be distinguished also between the string harmonics with the corresponding order of levels, i.e., the *ff* tone has typically the highest level and the *pp* tone the lowest. The behavior of the body modes between harmonics in general is not as clear as for the C_6 tone, but the trend is still notable.

The changes in body mode amplitudes for different dynamic levels are considerably larger than for the level variations in string motion. Hence, the changes in body mode amplitudes cannot be induced by the string vibrations only. Therefore, it seems that the with a forceful push of a key the soundboard and its modes are excited through mechanical coupling, separate from the string plucking mechanism. This mechanical excitation path and the mechanism from the key to the soundboard explains also why the body

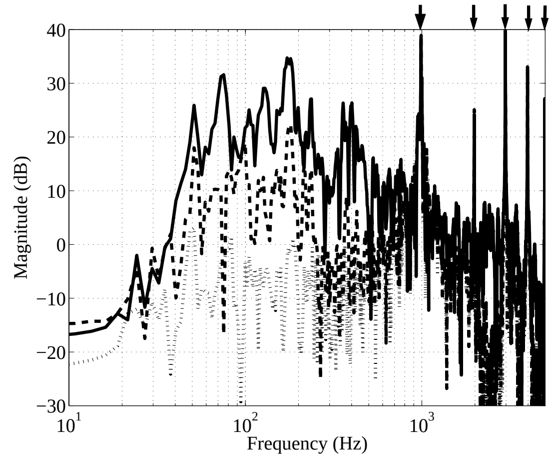

Figure 4: *Visualization of body mode behaviour for tone C_6 at different playing levels: ff (solid), mz (dashed), and pp (dotted). The string modes are indicated with arrows.*

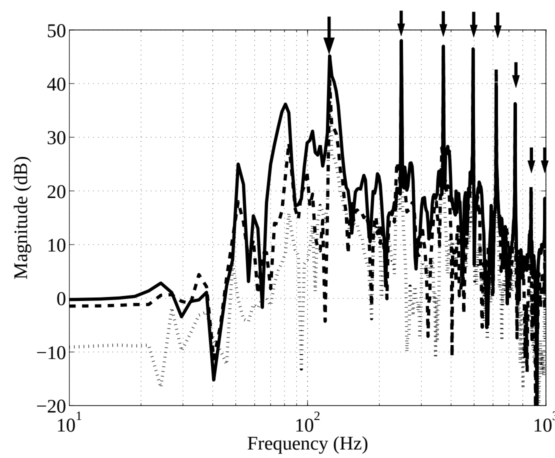

Figure 5: *Visualization of body mode behaviour for tone C_3 at different playing levels: ff (solid), mz (dashed), and pp (dotted). The string modes are indicated with arrows.*

modes have slightly different levels, Q-values, and frequencies in Fig. 4 and 5. This is because different excitation points along the keys excite the soundboard differently. In addition, coupling of body modes to the string modes can cause modes to be split or shifted in frequency.

4. LISTENING TEST

Computational models for the human auditory system exist, but they work the best for stationary sounds [15, 16]. So far, the reliability of auditory models is vague for complex tones, where the harmonic content and timbre change as a function of time. In addition, the loudest harpsichord tones exhibit the characteristic knock/thump of the soundboard in the attack as discussed in the previous section. This adds some ambiguousness to the *ff* and *mf* tones compared to the *pp* tones. Hence, a listening test is required to get to the bottom of the question on harpsichord dynamics.

[1]If the 50 Hz resonance were a mains-borne disturbance, the level would not increase with stronger playing levels.

A listening test was conducted where the perception of loudness of two tones was to be adjusted to be equal. Two recorded harpsichord tones were played consecutively, so that the gain of the latter sound could be adjusted in steps of 0.5 dB in a range of ± 4 dB. The adjusting was done with a slider and the tone pair could be listened to as many times as needed. The play list of the conducted test comprised of all pairwise comparisons between dynamic levels (*pp* to *ff*, *pp* to *mf*, and *mf* to *ff*) for all C tones (3x5). In addition, two blank pairs for each octave was added and the pairwise comparisons were also played in reverse order. All in all, the play list contained 40 pairs of tones.

The test was conducted in the listening room of the Acoustics Laboratory at Helsinki University of Technology. This high standard listening room has a very low noise level and it meets the ITU-R BS.1116 standard. The sounds were played through headphones to six participants with normal hearing. The sound level to the headphones was calibrated with the help of a dummy head and a pre-amplifier (Cortex electronic, Manikin MK1). The pre-amplifier had a sound pressure level (SPL) meter with A weighting and its values were matched with previously measured SPL values. The measured SPL values were obtained with an SPL meter (B&K 2238 Mediator with a 4188 capsule) placed near the ear of a player playing the same tones used in the analysis. These reference tones were measured with an Italian-type harpsichord in a rehearsal room at the Sibelius Academy. The *ff* C_4 tone was calibrated to 68.5 dBA.

Figure 6 shows the results of the listening test for each compared dynamic pair and all strings: (a) *pp* to *ff*, (b) *pp* to *mf*, and *mf* to *ff*. The x axis indicates the name of the tone and the y axis shows the perceived loudness difference in dB. In Fig. 6 each box has lines at the upper (75%) and lower (25%) quartile values. The median value is indicated between these values with the a line at the center of the hourglass-shaped part of each box. The whiskers (- - -) show the extent of the rest of the data. Outliers, indicated with a star (*), are data points with values beyond the whiskers.

For the results shown in Fig. 6, the largest mean value, 2.41 dB, was obtained for the C_3 tone with a standard deviation of 0.76, naturally found for the *pp* to *ff* case displayed in pane (a). Respectively, the smallest mean value 0.42 dB with a 0.7 dB standard deviation was obtained for the C_6 tone with a *pp* to *mf* tone pair. It seems that the dynamic differences are slightly more notable for C_3 and C_4 tones than for the lower or higher tones. For easy comparison, the mean values over all strings for each tested pair are given in Table 1. This way the tone-wise comparison is lost, but based on this it can be proposed that each dynamic step is perceivably different from the other. Furthermore, each dynamic step (*pp* to *mf* and *mf* to *ff*) is about the size of 1 dB and the dynamic range, at the largest, is slightly larger than 2 dB.

Based on the written and verbal feedback of the listening test, the testees mainly concentrated on listening to the decaying part of the tones. The knock of the soundboard present in the *mf* and especially in *ff* tones was noted and described as a characteristic that made the listening of loudness more challenging. This characteristic directed the listeners to focus on the decaying part of the tones.

The findings from the signal analysis discussed above, at least to some degree, support the results of the listening test, in the sense that the differences in the dynamics is smaller for higher strings than lower ones. On the other hand, the signal analysis results discussed in Sec. 3 could suggest quite large differences in the perceived loudness of the instrument. In the overall result, the ability of the soundboard to radiate efficiently and/or effects of the auditory system come into play. Consequently, the results from the listening test are quite understandable.

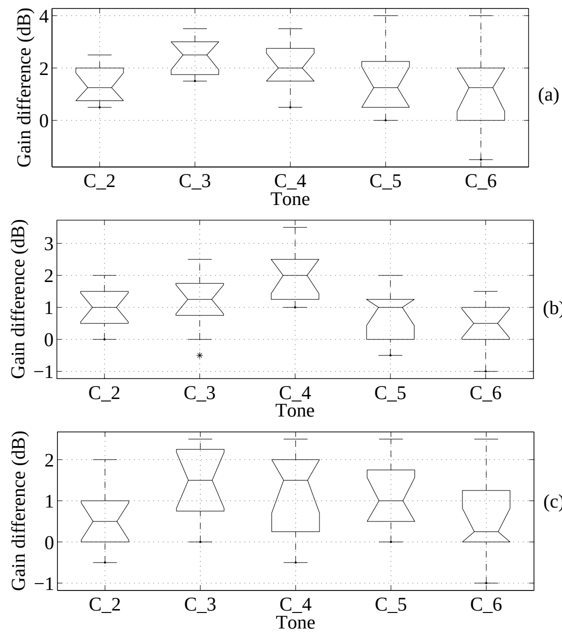

Figure 6: *Listening test results for tested tone pairs (a) pp to ff, (b) pp to mf, and (c) mf to ff. The x axis indicates the name of the tone and the y axis shows the perceived loudness difference in dB.*

Type	Mean	Std. deviation
pp to ff	1.68	0.37
pp to mf	1.08	0.09
mf to ff	0.98	0.18
blank	-0.45	0.24

Table 1: *Results from the listening test as mean values with standard deviation over all strings and for all three different transition types and the blank trials.*

5. SYNTHESIS MODEL WITH DYNAMICS

A general framework for modeling the dynamics of a harpsichord should include control over gain of string vibrations, frequency envelope of string vibrations, and simulation of the mechanically excited soundboard vibrations. These assumptions are based on the signals analysis and listening test discussed in Secs. 3 and 4.

Suppose that a model-based (physics-based) sound synthesis algorithm contains models for the excitation mechanism, the string vibrations, and the soundboard and radiation. Then, adding the modeling of dynamics would consist of a gain and timbre control for the string vibrations and an additional path from the excitation mechanism to a soundboard model bypassing the string model.

This train of thought is the basis for the synthesis model proposed here. The starting point for the synthesis model is a previously proposed model-based harpsichord sound synthesis algorithm [1]. This model [1] is founded on digital waveguide mod-

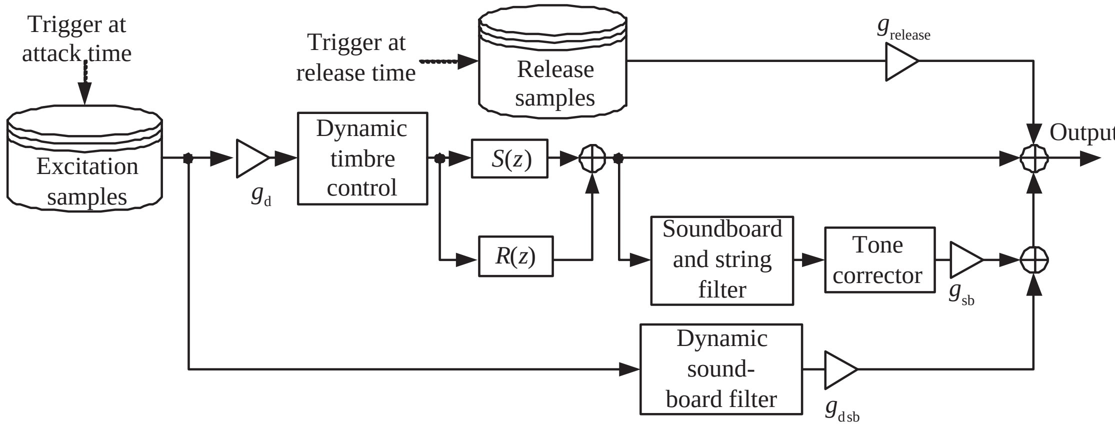

Figure 7: *Block diagram of the waveguide synthesis algorithm for the harpsichord with dynamics modeling.*

eling [17] and is a version of the commuted waveguide synthesis approach. The model already contains a so-called soundboard model/filter [1]. However, it is mainly intended for simulating the combination of soundboard modes and the slowly decaying string vibrations of the last octave of the 4′ register (without dampers) and the part of the strings that lies behind the bridge. These slowly decaying string vibrations dominate at frequencies above 350 Hz and their decay time T_{60} can be as long as 4.5 s. These vibrations are completely different from those examined in Figs. 4 and 5, which decay fast ($T_{60} < 0.5$ s). Therefore, the proposed synthesis model, shown in Fig. 7, contains an additional dynamic soundboard filter intended for simulating the soundboard knock audible in *mf* and *ff* tones. Moreover, since the origin of the soundboard knock seems to be the mechanical coupling from the tangent to the soundboard, the proposed model also bypassed the string model.

The excitation database for the harpsichord synthesizer is based on *mf* tones. The excitation signals are short bursts that are obtained by canceling the partials of the string vibrations with a sinusoidal model as discussed in [18]. To crudely model the higher levels of string harmonics the excitation fed to the string model is multiplied with g_{d} (see Fig. 7). To compensate for errors in the initial levels produced by the crude raising of the string vibration amplitudes with g_{d} the timbre control filter can be used. This naturally requires a higher order filter, and this necessity can be evaluated separately and can be simulated with any suitable parametric EQ filter (dynamic timbre control in Fig. 7). Finally, the behavior of the soundboard knock audible in *mf* and *ff* tones is simulated with the dynamic soundboard filter in parallel with the string model. The adjective dynamic is added since the filter has to change according to the kind of a dynamic level played, i.e., the filter coefficients are changed accordingly. Again, any proper parametric EQ-filter can be used. For modeling the transition from a *mf* tone to a *ff* tone the soundboard filter is required to boost the proper resonances, whereas in the case of a *mf* to *pp* transition, the soundboard filter attenuates the body modes.

Figure 8 shows the magnitude responses of a synthesized *mf* tone (dotted line), synthesized *mf* tone with a soundboard model (solid), target spectrum of the *ff* tone (dashed), and the soundboard model (solid), all for the C_6 tone. The soundboard filter is a cascade of six second-order parametric peak filters. The peak filters are discussed in [19](pp. 117-125). The body response of the synthesized *mf* tone with the soundboard model (solid) coincides

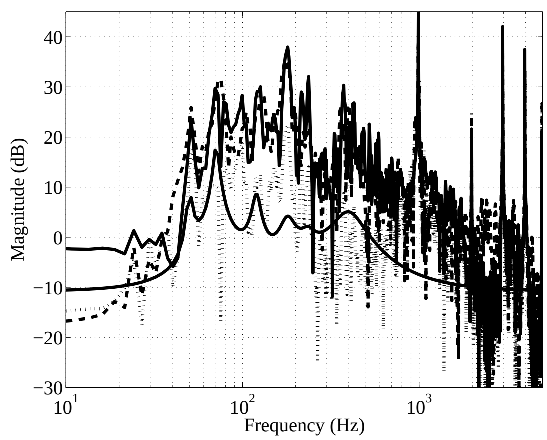

Figure 8: *Magnitude responses of* C_6 *tones: target ff (dashed), synthesized tone mf to ff (solid), synthesized mf (dotted), and soundboard filter (solid).*

quite well with the target *ff* tone (dashed) already with the cascade of only six peak filters.

6. DISCUSSION

Some of the reasons behind why the harpsichord exhibits dynamics and timbral changes have been covered; however, the final truth behind the string behavior still remains to be revealed.

A high speed camera would help out tremendously to give a more accurate estimate on how long the plucking event and contact between the plectrum and string lasts. This kind of data could enable to continue further the steps taken by Hall [7]. In addition, the question remains, does the release of the plectrum from the string occur before the reflected wave arrives back to the plectrum or after. A natural continuum from there on would be to investigate the interaction between the plectrum and the string when a guitar player is in charge. Another future path would be to investigate what happens in the harpsichord and to its loudness when two or more registers are played.

Even if the high frequency partials have relatively higher values when the dynamic level rises they decay significantly faster than the lower modes. This and the fact that the listener concentrates on the attack part could partly explain why the loudness differences were not larger in the listening test results. Longitudinal string modes or phantom partials [20, 21] are also present in the string vibrations, and they are clearly more pronounced for the *ff* tones than for *pp* tones. Their effect on the perceived loudness was, however, interpreted not to be as significant as the raising of absolute harmonic levels. They could, however, have an effect on the timbre of a tone.

7. CONCLUSIONS

Contrary to common assumption, the harpsichord contains a limited amount of dynamics and some timbral changes occur when the key is pressed down with different forces/speeds.

The signal analysis made on recorded harpsichord tones revealed differences in the levels of string harmonics, indicating stronger playing forces produced higher levels. The differences for isolated harmonics were as high as 5 dB for some low tones. For higher tones the level differences were smaller, about 1-3 dB.

Based on the conducted listening test, it can be said that during each dynamic step (from *pp* to *mf* and from *mf* to *ff*) the loudness of the instrument increases by about 1 dB. Furthermore, for the investigated harpsichord the dynamic range from *pp* to *ff* is at its largest for the C_3 tone with a value of slightly above 2 dB.

A synthesis model for approximating the dynamic behavior of the harpsichord is proposed. A general framework for building a model-based synthesis algorithm for the phenomenon is also given. Based on this, a specific model is proposed with gain and timbre control, and a parallel filter structure to simulate the soundboard knock.

The harpsichord exhibits dynamics albeit in a limited range, both in a musical and acoustical sense. However, it is doubtful that dynamic expressions will start appearing in harpsichord scores. Nevertheless, dynamic expressions can and should be included in the scores controlling a harpsichord synthesizer. What is more, the synthesizer can naturally take the dynamic range further than a real instrument can, as suggested previously [1]. Sound samples are available at the URL: http://www.acoustics.hut.fi/publications/papers/dafx2006-harpsichord-dynamics/.

8. ACKNOWLEDGEMENTS

Warm thank you to the participants of the listening tests, Hanna Järveläinen for her help, and to Jonte Knif for the intense discussions on the subject. This work has been financially supported by the Pythagoras Graduate School of Music and Sound Research.

9. REFERENCES

[1] V. Välimäki, H. Penttinen, J. Knif, M. Laurson, and C. Erkut, "Sound synthesis of the harpsichord using a computationally efficient physical model," *EURASIP J. on Applied Sig. Proc.*, vol. 2004, no. 7, pp. 934–948, 2004.

[2] N. H. Fletcher, "Analysis of the design and performance of harpsichords," *Acustica*, vol. 37, pp. 139–147, 1977.

[3] T. Gäumann, "The pretransient of the harpsichord sound. I. vertical movement of the plectrum," in *Proc. Stockholm Music Acoustics Conf. (SMAC'03)* Stockholm, Sweden, 2003, pp. 163–166.

[4] N. H. Fletcher and T. D. Rossing, *The Physics of Musical Instruments*. New York: Springer-Verlag, 1991.

[5] R. D. Weyer, "Time-frequency-structures in the attack transients of piano and harpsichord sounds - I," *Acustica*, vol. 35, pp. 232–252, 1976.

[6] A. H. Benade, *Fundamentals of musical acoustics*, 2nd ed. New York, NY, USA: Dover Publications, Inc., 1990. [Online]. Available: http://ccrma-www.stanford.edu/marl/Benade/

[7] D. E. Hall, "String excitation: piano, harpsichord and clavichord," in *Proc. Stockholm Music Acoustics Conf. (SMAC'93)* Stockholm, Sweden, 1993, pp. 309–314.

[8] M. Campbell and C. Greated, *The Musician's Guide to Acoustics*. New York, NY, USA: Schirmer Books, 1988.

[9] E. L. Kottick, "The acoustics of the harpsichord: Response curves and modes of vibration," *Galpin Soc. J.*, vol. 38, pp. 55–77, Apr. 1985.

[10] E. L. Kottick, K. D. Marshall, and T. J. Hendrickson, "The acoustics of the harpsichord," *Scientific American*, vol. 264, no. 2, pp. 94–99, 1991.

[11] W. R. Savage, E. L. Kottick, T. J. Hendrickson, and K. D. Marshall, "Air and structural modes of a harpsichord," *J. Acoust. Soc. Am.*, vol. 91, no. 4, pp. 2180–2189, 1992.

[12] R. D. Weyer, "Time-varying time-frequency-structures in the attack transients of piano and harpsichord sounds - II," *Acustica*, vol. 36, no. 4, pp. 241–258, 1976.

[13] A. Beurmann and A. Schneider, "Sonological analysis of harpsichord sounds," in *Proc. Stockholm Music Acoustics Conf. (SMAC'03)* Stockholm, Sweden, 2003, pp. 167–170.

[14] J. Sankey and W. A. Sethares, "A consonance-based approach to the harpsichord tuning of Domenico Scarlatti," *J. Acoust. Soc. Am.*, vol. 101, no. 4, pp. 2332–2337, 1997.

[15] E. Zwicker and H. Fastl, *Psychoacoustics: Facts and Models*. Berlin: Springer-Verlag, 1990.

[16] B. C. J. Moore, B. R. Glasberg, and T. Baer, "A model for the prediction of thresholds, loudness and partial loudness," *J. Audio Eng. Soc.*, vol. 45, pp. 224–240, 1997.

[17] J. O. Smith, "Physical modeling using digital waveguides," *Computer Music J.*, vol. 16, no. 4, pp. 74–91, 1992, [Online] http://www-ccrma.stanford.edu/~jos/pmudw/.

[18] V. Välimäki and T. Tolonen, "Development and calibration of a guitar synthesizer," *J. Audio Eng. Soc.*, vol. 46, pp. 766–778, 1998.

[19] U. Zölzer, *Digital Audio Signal Processing*. Chichester, UK: J. Wiley & Sons, 1997.

[20] H. A. Conklin, "Generation of partials due to nonlinear mixing in a stringed instrument," *J. Acoust. Soc. Am.*, vol. 105, no. 1, pp. 536–545, 1999.

[21] B. Bank and L. Sujbert, "Generation of longitudinal vibrations in piano strings: From physics to sound synthesis," *J. Acoust. Soc. Am.*, vol. 117, no. 4, pp. 2268–2278, 2005.