REMOVING CRACKLE FROM AN LP RECORD VIA WAVELET ANALYSIS

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ABSTRACT

The familiar "crackling" is one of the undesirable phenomena which we deal with in an LP record. Wavelet analysis brings a new alternative approach to the removal of this feature in the restoration process of the recording. In the paper, the principle of this method is described. A theoretical discussion of how the selection of the wavelet basis affects the quality of the restoration is also included.

1. INTRODUCTION

The wavelet-type signal analysis has recently been a much used discipline, whose range of applications in one-dimensional and multi-dimensional signal processing spreads steadily. It is used for the time-frequency analysis, for reconstruction of non-complete or strongly disturbed signals and for data compression in many fields.

There were many algorithms developed how to restore the digitized audio signal from a vinyl record, for example SDROM (Signal Dependent Rank Order Mean) [3], methods based on linear prediction in AR and ARMA models [6] or the well-known median filtering. Another special class of restoration methods exploit the Bayesian statistics [5]. This paper introduces an alternative approach to the problem, using the wavelet signal processing.

In the paper, we first discuss the time and frequency characteristics of a "crackle". After this an overview of wavelet transform and its properties necessary for the method's derivation is presented. Then, the principle of the crackle removal is described. At the conclusion, the discussion of how the choice of the so-called mother wavelet affects the quality and effectiveness of the restoration is introduced.

1.1. Characterization of a "crackle"

The typical behavior of the crackle in the time domain usually corresponds to signal waveform "up and down". We found that three quarters of these peaks stretch from 0.36 to 1.09 ms. The crackle has also bigger short-time energy than the rest of the signal. Another typical feature is that in spectral domain it spreads over all the frequency bands (see Figure 1). The character of the crackling also slightly differs depending on which part of the recording we work on – this is due the different speed of vynil rotation at the beginning and at the end of the recording.

2. WAVELET TRANSFORM

We will present some necessary basics of wavelet transform, first on signals with continuous time, after that we will switch to the discrete-time wavelet processing.

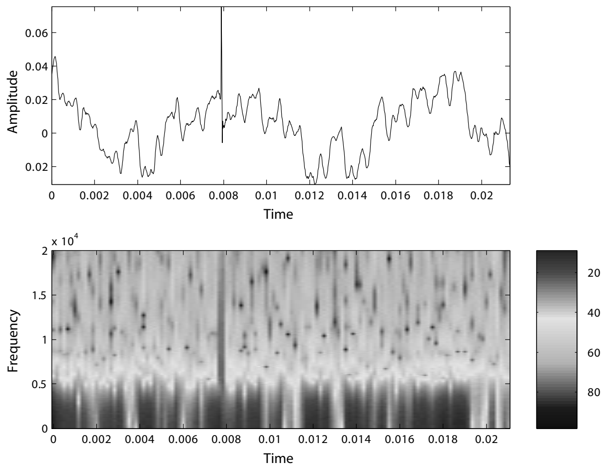

Figure 1: *The time behavior and the spectrogram of the crackle. The crackle is spread all over the frequencies.*

In processing signals with continuous time (i.e. functions) wavelet transform means a signal's decomposition into countably many "atoms" – wavelet functions – which are all derived from just one function, called scaling function [2, 4], and which form a signals' space basis.

The discrete wavelet transform (but still in continuous time!), DWT, of a real $f(t)$ is the set of so-called wavelet coefficients $c_{j,k} = \int_{-\infty}^{\infty} f(t)\,\psi_{j,k}(t)\,\mathrm{d}t, j,k \in \mathbb{Z}$, where we define $\psi_{j,k}(t) = 2^{j/2}\psi\left(2^j t - k\right)$ and $\psi(t)$ is a "mother wavelet". There are conditions that ψ must satisfy and from them we can conclude that ψ must have zero mean (thus it is of oscillatory character), and that the oscillations must necessarily attenuate towards $\pm\infty$.

An arbitrary signal $f(t)$ then can be expressed as the linear combination of basis wavelets $\psi_{j,k}$:

$$f(t) = \sum_{j,k\in\mathbb{Z}} c_{jk}\,\psi_{j,k}(t). \qquad (1)$$

2.1. Multiresolution analysis

Small values of j in the above expansion correspond with the trend contained in the signal, whilst for $j \to \infty$ basis functions $\psi_{j,k}$ convey the signal's behavior in more detail. This feature enables us to decompose an arbitrary signal, even with discontinuities or sharp peaks. This type of decomposition is called multiresolution analysis (MRA). An example of such analysis can be found in Figure 4.

2.2. Compact-supported wavelets

There were even found orthonormal bases such that their elements have compact support, i.e. there can be found a closed interval such that outside it the basis function is zero. An example of such functions can be the Daubechies-type wavelets [1].

The compactness of the wavelet's support plays an important role for our purposes of "decrackling". The fact that a wavelet ψ vanishes outside a close interval means that every translation and dilation $\psi_{j,k}$ contributes to the signal just *locally*.

2.3. Wavelets with vanishing moments

Vanishing moments is another important concept in wavelet signal processing. A wavelet is said to have k vanishing moments if

$$\int_{-\infty}^{\infty} \psi(t)\, x^i = 0 \quad \text{for } i = 0, 1, \ldots, k-1. \tag{2}$$

Equation (2) can be interpreted as follows: if ψ has k vanishing moments, then every polynomial of order $k-1$ or less can be represented as a linear combination of the scaling function. It means that in MRA of the polynomial-like signal all the coefficients representing the signal's details will be zero. This is again an important feature for our purposes, because if there was a single, fast crackle in an audio signal, which we consider as locally polynomial, this would lead to non-zero detail coefficients right in place of the singularity.

2.4. Discrete-time wavelet analysis

In practical problems we most frequently work with discretized (sampled) signals of finite length. In this case we speak of the finite discrete wavelet transform (DTWT), which can be represented by an orthogonal matrix $\mathbf{W}$ of size $n \times n$. Let $\mathbf{x} = [x_1, \ldots, x_n]^\top$ be a vector of length n. Its wavelet transform is vector $\mathbf{y} = [y_1, \ldots, y_n]^\top$, obtained as $\mathbf{y} = \mathbf{W}\mathbf{x}$. Due to the orthogonality of $\mathbf{W}$, the inverse wavelet transform is $\mathbf{x} = \mathbf{W}^{-1}\mathbf{y} = \mathbf{W}^\top\mathbf{y}$. It is evident from the above text that the wavelet transform has an important property – *linearity*.

Instead of multiplying vectors $\mathbf{x}$ and $\mathbf{y}$ by orthogonal matrices $\mathbf{W}$ and $\mathbf{W}^\top$, respectively, more effective Mallat's pyramid algorithm [4] is used for computing the transform. Each step of this algorithm corresponds to filtering a discrete series by specific low-pass and high-pass filters and then decimating the result. The coefficients from the low-pass branch are called "approximations" and those from the high-pass branch are called "details". We can repeat this single transformation step with the approximations standing for the input signal. The number of repetitions is called transformation depth. Scheme of this algorithm is depicted in Figure 2.

This way the input is divided into a number of subbands. Figure 3 shows the idealized decomposition in frequency domain.

The algorithm of the inverse wavelet transform is similar: we pass through the decomposition "tree" in the opposite direction performing reverse operations.

Wavelets differ by the decomposing and reconstruction filters. The filters corresponding to the compact-supported wavelets are always FIR filters. The coefficients of the filters determine their frequency response and thus the quality of signal splitting into

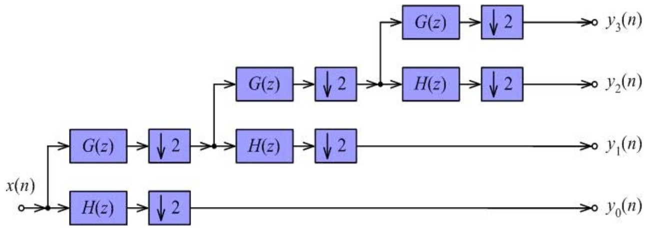

Figure 2: *Mallat's algorithm of wavelet transform. The input signal $\mathbf{x}$ is decomposed into its wavelet coefficients, contained in $\mathbf{y}_0, \ldots, \mathbf{y}_3$. The decomposition depth is 3.*

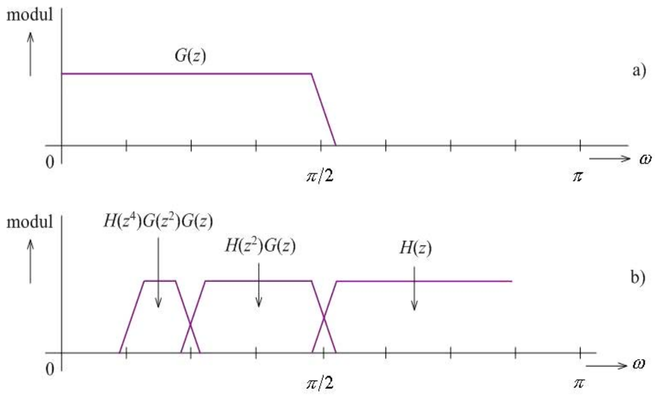

Figure 3: *Idealized subband decomposition of the input signal. $H(z)$ represents the frequency response for the high-pass filter, $G(z)$ represents the response for the low-pass.*

frequency subbands. Generally, the sharper slope is required, the more coefficient are needed.

3. PRINCIPLE OF THE WAVELET-TYPE PROCESSING OF CRACKLES

Our wavelet method starts from the assumption of addivity of the disturbing crackles. This means that the impulses are added to the signal which we desire to restore. Formally,

$$\mathbf{y} = \mathbf{x} + \mathbf{p}, \tag{3}$$

where $\mathbf{x} = [x_1, \ldots, x_n]^\top$ is the original music or speech signal without any undesirable artifacts, $\mathbf{p} = [p_1, \ldots, p_n]^\top$ is the random "crackling" signal, and $\mathbf{y} = [y_1, \ldots, y_n]^\top$ is the mixed signal, which we have observed.

Starting from this model, we can make following inference. As said above, the wavelet transform is linear, and thus it holds

$$\mathbf{W}\mathbf{y} = \mathbf{W}\mathbf{x} + \mathbf{W}\mathbf{p} \tag{4}$$

for the observed signal $\mathbf{y}$. Then for the original "clean" signal in the wavelet domain there must be $\mathbf{W}\mathbf{x} = \mathbf{W}\mathbf{y} - \mathbf{W}\mathbf{p}$. Applying the inverse transform via matrix $\mathbf{W}^{-1} = \mathbf{W}^\top$ we ideally obtain the desired signal without any corruption:

$$\mathbf{x} = \mathbf{W}^{-1}(\mathbf{W}\mathbf{y} - \mathbf{W}\mathbf{p}). \tag{5}$$

Our method works on the principle formally stated in (5):

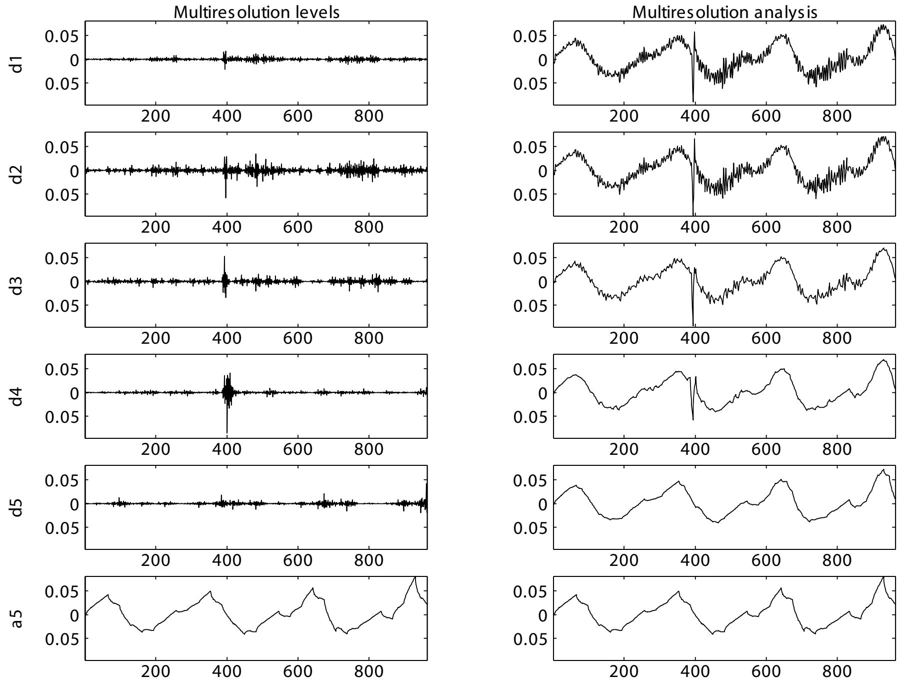

Figure 4: *Multiresolution analysis of a signal with wavelet Daubechies of order 2. In the left column there are plots of contributions of single signal subspaces and in the right column there are their respective cumulative sums. It is clear that some subspaces contain more aproximate view of the signal and others contain more details. The most up-right picture is the original input signal.*

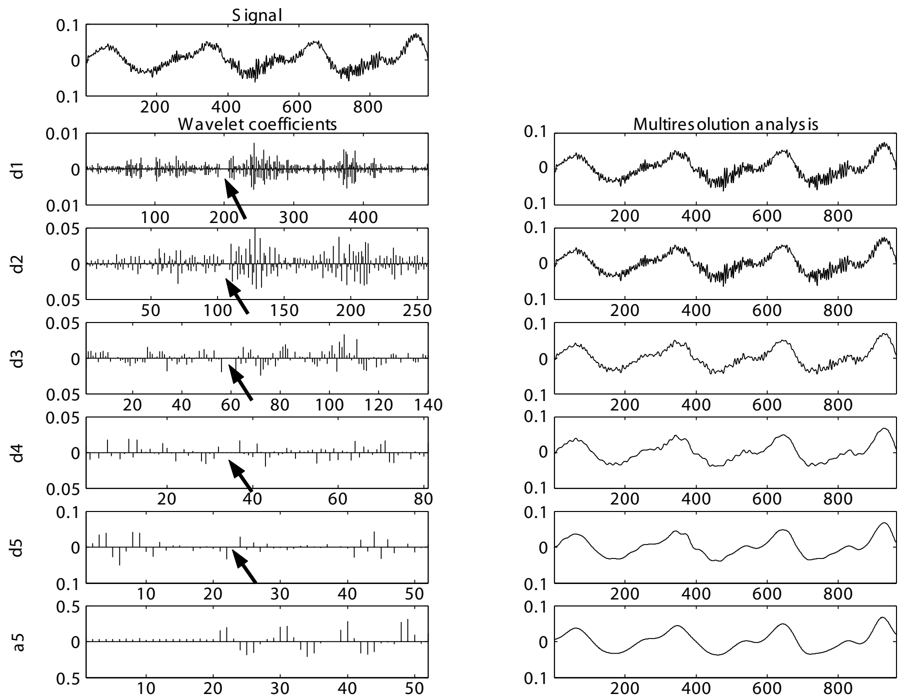

Figure 5: *Restoring the recording via the wavelet algorithm. Daubechies of order 12 was used for the multiresolution analysis. The arrows indicate the detail wavelet coefficients that were set to zero. The output signal is the most upper one. The input signal was the same as the one in Figure 4.*

We regard the detail wavelet coefficients in the close neighbourhood of a crackle as its transform, i.e. $\mathbf{Wp}$. Setting them to zero (on principle we perform "anti-thresholding") we locally suppress the contribution of the detail levels in the multiresolution analysis right in the place where the impulse was detected. Speaking of frequency domain, we locally and *in gradual degree* pass low frequencies in place of the impulse and suppress the high frequency components. The graduality is reached due to the fact that the support of the wavelet filters doubles in each multiresolution level. From the crackle position onwards, naturally, we again pass gradually more and more high frequencies. The described process lasts a bit longer than the crackle does, i.e. about 1 ms.

Figure 5 shows a successful application of the algorithm.

To be more precise, the algorithm consists of two parts, detection and elimination. Each part can use different wavelet. The main steps of the two parts are:

detection

1. signal transform with wavelet chosen for detection
2. passing through the detail levels and marking places suspected of impulse (based on an energetic criterion)
3. comparing detail coefficients belonging to these places with a properly set threshold
4. if the last step confirms there are detail coefficients above the threshold in the same place, the center of it becomes the center of the area to be modified. The width of this area depends on the amount of the coefficients above the threshold.

elimination

1. signal transform with wavelet chosen for elimination
2. detail coefficients belonging to the area specified above are set to zero, whereas in every subsequent decomposition level there are approximately half of the coefficients processed in the preceding level. This is due to the decimation step of the wavelet transform.
3. inverse wavelet transform

For better performance, the algorithm could be run recursively, i.e. multistage.

4. FACTORS DETERMINING THE QUALITY OF RESTORATION

In this Section, we discuss factors that affect the restoration quality.

The compactness of the wavelet support plays an important role. This is because the crackle is just a local artifact and can be expressed by only few wavelet coefficients which refer directly to the place where the impulse is situated. Thus, it is effective to use wavelets with compact support (Daubechies, Symlets etc.), which correspond to FIR filters.

Another important property of a wavelet is the above mentioned number of vanishing moments. This number is closely associated with the number of continuous derivatives (smoothness) and in the discrete-time processing, it is directly connected to the sharpness of the filters' slopes. The more vanishing moments a wavelet has, the better is the signal frequency separation. Because for the purposes of decrackling we don't want the frequency bands to soak much into each other, it is better to use wavelets with more vanishing moments, i.e. wavelet filters of a bigger order. For example, see Figures 4 and 5 – comparison of the two multiresolution analyses domonstrate the difference in decompositions' smoothness. We have found that Daubechies wavelets of order about 10 give sufficient frequency separation, in proportion to the computational complexity.

The last factor affecting the restoration quality is the transform depth. By selecting inproper transform depth, we could make an insufficient number of frequency subbands and thus not separate the crackle from the rest of the signal. For audio signals sampled at 44.1 kHz we found that choosing depth 5 or 6 is sufficient, i.e. it is adequate to process 5 or 6 sets of detail coefficients, which means we leave frequencies below 800 Hz intact.

5. PERFORMANCE OF THE METHOD

In the core of wavelet transform stands correlation of the signal with the dilated and translated wavelets. Due to the fact that the wavelets are of strongly oscillatory character, our method turned out to be successful for suppressing crackles of this character – attenuated oversights up & down. However, there also often occur crackles that do not observe this character, and for these, the algorithm's performance is naturally worse.

6. CONCLUSION

In the paper, a new method of vynil record restoration was introduced. The method is based on the wavelet-type signal analysis and forms an alternative to the commonly used methods. The crackling is suppressed via local processing of the so-called detail wavelet coefficients. There is also a discussion what factors determine the quality of the restoration process.

7. ACKNOWLEDGEMENTS

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