DIGITAL GUITAR BODY MODE MODULATION WITH ONE DRIVING PARAMETER

Henri Penttinen, Aki Härmä, and Matti Karjalainen

Helsinki University of Technology, Laboratory of Acoustics and Audio Signal Processing
P.O. Box 3000, FIN-02015 HUT, Finland
henri.penttinen@hut.fi, aki.harma@hut.fi, matti.karjalainen@hut.fi
http://www.acoustics.hut.fi/

ABSTRACT

In this study we have developed a digital guitar body mode modulation technique where the modulation can be controlled through one driving parameter. The filtering and modulation is done with frequency-warped recursive filters that have been implemented in real-time on a modern DSP processor. By changing the warping parameter the perceived size of the body can be controlled, by a pedal or automatically, resulting in an interesting effect. This effect is useful both for the electric and the amplified acoustic guitar. Perceptual properties of the effect are studied by a listening experiment. (See also www.acoustics.hut.fi/demo/dafx2000-bodymod/)

1. INTRODUCTION

Digital filters have been successfully used to model the body of string instruments [1], [2], [3]. Usually relatively high-order filters are needed, e.g., in modeling of acoustic guitar body. In [4] it was shown that it is beneficial to model the guitar body using frequency-warped filters [5], [6], [7]. The order of a warped filter may be significantly lower than the order of a corresponding conventional filter. This is due to the fact that design of warped filters utilizes a modified frequency representation. This altered frequency representation matches better to the resonances of a guitar body and properties of human hearing [7] than conventional designs and filters which use uniform frequency representation. The frequency-warping effect in the filter is controlled by a single parameter λ. It was already pointed out by Lansky and Steiglitz in [8] that when a warped filter is designed to match with the resonance structure of a particular instrument, the use of the filter with a 'wrong' warping parameter yields an approximation for the resonance structure in a similar but smaller or larger instrument.

There are two goals in this paper. First, we try to verify the assumption of Lansky and Steiglitz in the case of a carefully designed acoustic guitar body model which is driven by a guitar pickup signal. This is done by spectral analysis and listening tests. Both an electric guitar and an acoustic guitar can be used as sources for the excitation signal. Secondly, we introduce and study a new guitar effect where the parameter is controlled continuously to produce an impression of an acoustic guitar having a highly flexible time-varying body.

2. FREQUENCY-WARPED FILTERS

Warped digital filters and their audio applications have been introduced and discussed in more detail elsewhere [9, 6, 10, 11, 7]. Only a short overview is given here.

The basic idea of warping is best illustrated using the filter structure in Fig. 2a. When the unit delays of an ordinary digital filter are replaced with first-order allpass sections, the resulting filter is called a warped filter. The filter can be designed on a warped frequency scale based on the bilinear conformal mapping

$$\tilde{z}^{-1} = D_1(z) = \frac{z^{-1} - \lambda}{1 - \lambda z^{-1}} \tag{1}$$

where λ is a warping parameter and $D_1(z)$ is a warped delay element. The group delay of $D_1(z)$ is frequency-dependent so that positive values of λ yield increased resolution at low frequencies. Correspondingly, a negative value of λ produces a system with an improved frequency resolution at high frequencies. This is illustrated in Fig. 1 that shows the warping by a first-order allpass section as a function of frequency. The value of the warping parameter should be $|\lambda| < 1$ to make the allpass filter stable. The warping parameter λ is the *one driving parameter* mentioned in the title of this article.

Design and implementation of warped transversal (WFIR) structures is straightforward. However, the implementation of warped recursive filters is problematic because there are delay-free loops in the filter. Implementation techniques for warped recursive direct-form and lattice filters have been introduced in [12]. There are two approaches: direct implementation of the filters using a specific two-step algorithm, and elimination of the delay-free loops by modification of the filter structure. A modified structure is shown in Fig. 2b. The new coefficients σ_k of the filter can be computed using an algorithm presented, e.g., in [11]. This filter structure is used in the current article, although it would be computationally slightly more efficient to use the direct implementation of the original filter if λ changes at each sample.

As noted above, the value of the warping parameter λ controls the amount of warping that is desired. From the point of view of auditory perception a specific value of λ yields a good approximation of the Bark scale [13] which is traditionally used as a psychoacoustical pitch scale. A formula to compute this value as a function of the sampling rate is given by Smith and Abel in [14]. At the sampling rate of 48 kHz this yields $\lambda \approx 0.76$. In the current article we use $\lambda = 0.73$. This value was found to enable a larger range for the modulation of the parameter.

One more favorable property of warped filters is their inherent robustness in precision requirements, based on the use of allpass subsections. Particularly, when the density of poles and/or zeros in the z-domain—especially corresponding to low frequencies—is high, traditional filter structures such as direct form IIR filters become very problematic. Due to the bilinear warping (rotation) of poles and zeros in the z-domain, the pole and zero densities

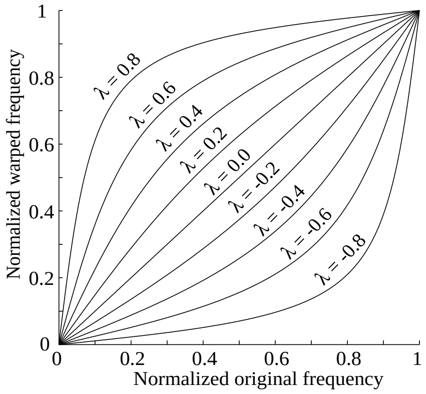

Figure 1: *Frequency warping characteristics of the first-order allpass section for different values of the warping parameter λ. Frequencies are normalized to the Nyquist rate.*

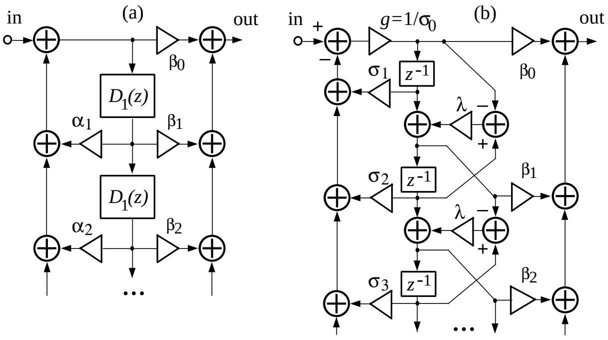

Figure 2: *(a) The principle of warped IIR filter and (b) the implementation used in this study.*

are relaxed considerably. Typically, direct form IIR filters higher in order than about 20–25 cannot be implemented even when using floating-point processors. Corresponding warped filters remain stable and realizable even with orders higher than 100 and fixed-point computation [15].

As can be noticed when comparing the warped IIR filter in Fig. 2b with traditional IIR filter structures of the same order, the warped structure is more complex and thus computationally more expensive. This is often compensated, however, since considerable reduction in filter order is possible due to good match to human auditory frequency scale properties.

The design of warped filter models for a measured impulse response can be done in a straightforward way as follows. The measured impulse response $h(n)$ to be implemented is first mapped to a warped time domain response $\tilde{h}(i)$ using the inverse mapping of (1) as described in [11]. This warped impulse response can then, in principle, be used in any filter design technique to yield an FIR or IIR structure, which has to be implemented as a corresponding warped structure. Furthermore, to create a warped model, the warping coefficient, λ, used in the warped structure is the same as the one used at design stage of the filter. For warped designs we have successfully used the warped toolbox for Matlab available in [16]. Further details of warped guitar body modeling are discussed later in this article.

3. FILTER DESIGN FOR BODY SIMULATION

The response of a pickup attached to the bridge of an acoustic guitar is inherently different from the acoustic radiation in the near or far field of the guitar. This is due to the incapability of the pickup to capture the influence of the guitar body or the direct string radiation. The guitar body (soundbox) affects the audible response by amplifying the string vibration and by giving it a reverberant and colorized response. There are typically at least two strong modes at low frequencies ($\approx$ 80-200Hz) and at higher frequencies the modes are weaker and their density is higher. The transfer function from the bridge vibration to the radiated sound can be approximated as a linear and time-invariant (LTI) system [1], [2], and [3]. Therefore, by processing a bridge pickup signal with a properly designed digital filter, a response that is an approximation of the radiated sound is rendered. The filter design methods introduced shortly here are discussed more thoroughly in [17] and [18].

An equalization (EQ) filter that simulates the missing body can be calculated through frequency-domain convolution as follows

$$h_{eq}(n) = FFT^{-1}\left(\frac{FFT[p(n)]}{FFT[x(n)]}\right), \qquad (2)$$

where *p(n)* and *x(n)* are the acoustic response and the bridge pickup response, respectively, *FFT* is the fast Fourier transform, and *n* is the discrete time-variable. The synchronous signals *p(n)* and *x(n)* are captured in an anechoic chamber with a microphone, placed 1 m in front of the soundbox, and a bridge pickup. The described setup is suitable for flat-top and classical guitars which use steel and nylon strings, respectively.

For an electric guitar a similar body simulation filter can be derived. In this case, there are two alternatives how to acquire it. In the first one, a modified impulse response of an acoustic guitar is used. Here the lowpass filtering effect of the magnetic pickup [19] has to be compensated by highpass emphasis. The impulse response is measured in an anechoic chamber by hitting the bridge of an acoustic guitar. Both classical and flat-top guitars can be used in this method. In the second method a variation of the measurement setup described for the acoustic guitar is used. A magnetic pickup, for *x(n)*, is mounted to the soundhole of an acoustic guitar, and a measurement microphone, for *p(n)*, is set 1 m in front of it. Similarly as before, the two signals can be deconvolved in the frequency domain (Eq. 2), and a desired EQ filter is attained. In this second method, applied for the electric guitar, only flat-top guitars can be used, since the nylon strings of a classical guitar do not induce voltage to the magnetic coil, like steel strings do.

Once a guitar body simulation filter has been designed, a warped model of it can easily be created. In this study, we have warped a minimum-phase impulse response of a soundbox filter. The motivation for using a minimum-phase impulse response, rather than the impulse response itself, is a consequence of temporal properties of the warped structure. It will say, the high frequencies of a warped non-minimum-phase impulse response will become excessively long in the time-domain. While using a minimum-phase impulse response to create an all-pole model it is adequate to apply linear prediction (LP), rather than using for example the Prony's method. The transfer function of the warped all-pole body-model is the following

$$H(z) = \frac{1}{A(z)} = \frac{1}{1 + \sum_{k=1}^{N} \alpha_k D_1(z)^m}, \qquad (3)$$

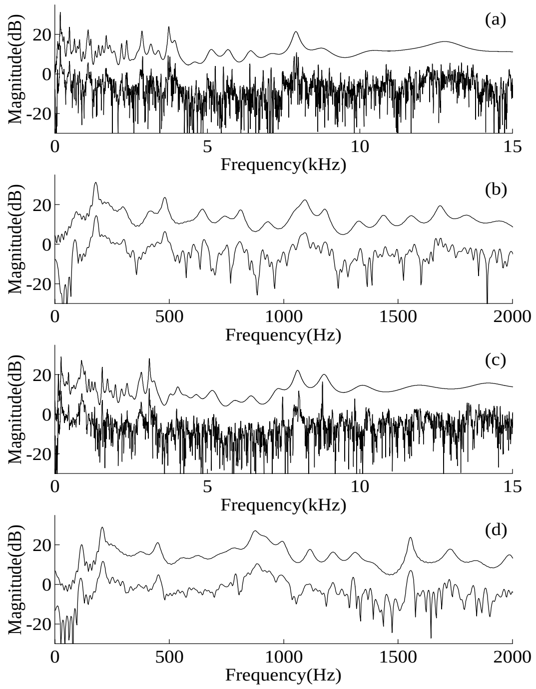

Figure 3: *Magnitude responses for FIR and WIIR body simulation filters. Panes (a) and (b) illustrate a large body filter and panes (c) and (d) a small one. For a more illustrative representation, the FIR responses are depicted beneath the WIIR ($\lambda = 0.73$) responses.*

where α_k represents the LP coeffients, of order *m*, and D_1 is the dispersive delay element of Eq. 1.

As stated in [17], FIR body filters of order 1000 are sufficient to give a pleasing result, but to model the lower body modes better, a WIIR model was derived from a 3000 taps long impulse response.

By using an all-pole WIIR model of order 100, designed as described above, the important characteristics of the body-model are conserved. However, some reverberant characteristics at high frequencies are lost due to concentration on the lower frequencies through warping. On the other hand, without warping the lowest resonances would not be modeled properly. Figure 3 illustrates magnitude responses of warped (WIIR) and unwarped (FIR) body-models. In Fig. 3a and 3b the spectra correspond to a large soundbox filter, and 3c and 3d with a small soundbox filter. In each pane, from (a) to (d), in Fig. 3, the WIIR filters are depicted above the equivalent FIR filters for a more clarified illustration. In panes (b) and (d) of Fig. 3, a zoomed view of the lower frequencies is presented, to show how well the lowest resonances are modeled by the WIIR filters. The two lowest body modes of the larger soundbox filter are situated at 87 Hz and 181 Hz, and the modes of the small body filter are at 112 Hz and 210 Hz. The FIR filters are 3000 taps long and the order of the WIIRs is 100.

4. FILTER MODULATION

In this study we let the warping parameter, λ, be a free parameter which can be adjusted freely in real-time. This produces a non-uniform spectral modification where all resonances and anti-resonances are shifted non-uniformly up or down in the frequency domain.

The warped implementation structure will resolve in a low or high-boost filtering effect, which is dependent on the value of λ. This spectral tilt effect can be compensated by pre-filtering the input signal before the body model filter, with the following filter

$$H_c(z, \lambda_m) = \frac{\sqrt{1-{\lambda_m}^2}}{\sqrt{1-{\lambda_o}^2}} \frac{1-{\lambda_o}^2 z^{-1}}{1-{\lambda_m}^2 z^{-1}}, \tag{4}$$

where λ_o is the original warping parameter that was used to warp the minimum phase impulse response, and λ_m is the modified value of λ that will shift the resonances.

The value of the warping parameter, λ, can be altered in a predetermined way or for example with a control pedal. This way an interesting audio effect is attained that can, at a principle level, be used at least in two ways: (I) One can maintain a particular λ value for a musically meaningful period (e.g., a riff or two, or a whole song) and then change it to another value. This way the impression of different sized guitars with the same instrument can be achieved. (II) By continuously adjusting the value of λ, a steadily changing timber will be observed. In this case identifying various sized soundboxes is almost impossible, but adds a pleasant and useful effect to the 'effect-toolbox' of guitarists. As mentioned before the spectral tilt effect caused by warping has to be reversed with Eq. 4. This in mind the body-modulation effect can be expressed in the z-domain as

$$\tilde{x}(z) = H_c(z, \lambda_m) \frac{1}{A(z)} x(z) \tag{5}$$

where *x(z)* is the input signal, $H_c(z, \lambda_m)$ is the compensation filter, *A(z)* is the body-model, and $\tilde{x}(z)$ is the output that has been transformed to something novel by means of warping. In real-time implementation, the mapping of the body-model coefficients (α_k to σ_k) can be performed for every sample or spread over a few samples. In most trivial warped FIR form the presented technique is also analogous to the traditional digital phaser effect [20]. The highly flexible usage of λ also perceptually resembles the phaser effect.

Figure 4 displays a set of magnitude responses with different λ values from 0.65 to 0.81 in steps of 0.01. The magnitude responses are stacked, one above the other, so that the λ value that corresponds with a spectrum is displayed on the y-axis. Figure 4 illustrates how the resonances shift when the value of λ is altered. The initial non-warped magnitude response is displayed in the middle of Fig. 4, with λ = 0.73. All the warped magnitude responses, i.e., $\lambda \neq 0.73$, are compensated with $H_c(z)$.

Figure 5 illustrates the highly flexible behavior of the body-modeling filter, when the warping coefficient, λ, is modified beyond a recognizable guitar body. Figure 5 shows magnitude responses of a body simulation filter as a function of the warping coefficient λ with steps of 0.01 from -0.756 to 0.756. The frequency axis is linear (vs. Fig. 4). Dark colors in Fig. 5 represent strong resonances of the body simulation filter.

5. PERCEPTUAL ASSESSMENTS

Lansky and Steiglitz proposed that by warping an LP-model, used for synthesizing a digitized violin piece, different instruments of a violin family can be created. The LP models they used were derived from digitized violin playing. As the excitation for a single LP-model they used a roughly triangular waveform, with a certain frequency, and warped the LP-model in a way that corresponds to

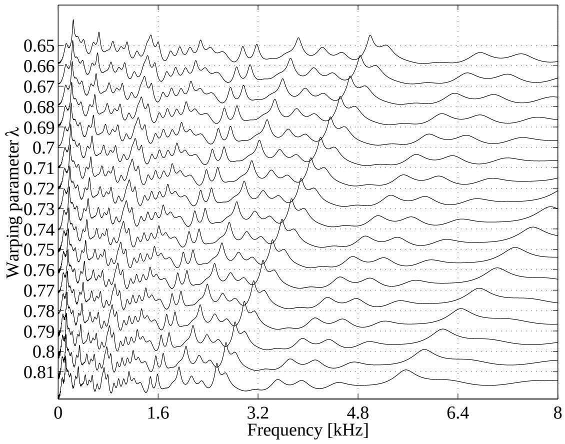

Figure 4: *Body simulation filter's magnitude responses with different warping parameter, λ, values.*

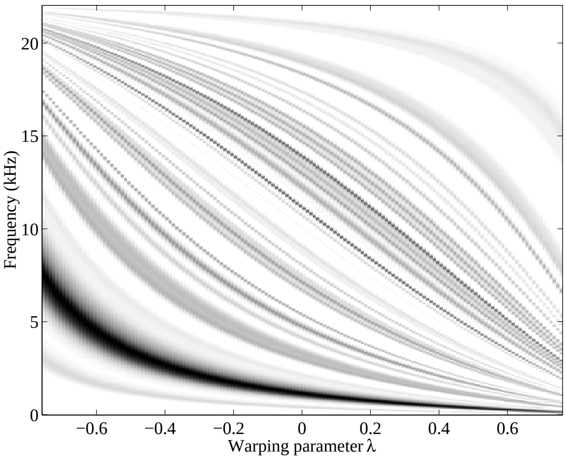

Figure 5: *Body simulation filter's magnitude response as a function of the warping parameter λ. The tone values indicate the magnitude of the guitar body resonances. Darkest colors represent the strongest resonances.*

the tuning of the targeted instrument. In this work we alter only the body simulation filter and leave the original excitation signal untouched. The aim is to create an effect, to be applied to the signal from a guitar pickup, that will change the size of the perceived soundbox. As one part of this study, we try to verify the assumption of Lansky and Steiglitz in the case of a carefully designed acoustic guitar body model which is driven by a guitar pickup signal, through listening tests and spectral interpretation.

5.1. Description of the Listening Test

In the conducted listening test the task of the testee was to size two perceived soundboxes as closely to one another as possible, by adjusting the warping parameter, λ. A reference signal and a test signal were played, where the latter was to be matched with the previous one. The pair of signals were filtered with different body filters, derived from small and large sized guitars. When the reference signal was processed with a large body-model the test signal was filtered with a small soundbox filter, and vice versa. In the test, the pair of signals was played in a continuous loop, with a short break between each sound sample. When the subject was satisfied with the matching of one pair, the testee proceeded to the next one. The warping parameter could be adjusted within steps of 0.001, from 0.53 to 0.93. The control of λ and the transition to the next sound sample pair was implemented through a graphical interface. The listening test was carried out by seven musically oriented subjects with normal hearing, in a standard listening room by using headphones.

Guitar playing, a short impulse train, and a burst of white noise were used as the excitation signals. Furthermore, there were two kinds of melodic guitar samples: one consisted of the instrument excited by picking and the other one was excited by strumming. All samples were filtered with WIIR filters of order 100. During the try out of the listening test it was noticed that altering λ shifted the resonances in a desired way. But as a side-effect, a distinct timbre emphasis was perceived, which altered with the adjustment of λ. This was considered subtly disturbing and viewed as a characteristic that might complicate the matching task. To diminish the timbre emphasis the resonances were broadened in the frequency-domain, i.e., shortened in the time-domain. This was done by smoothing the LP body-model coefficient sets (both large and small body filters) in the following way

$$\tilde{\alpha}_k = \alpha_k a^k; k = [0, 1, 2, ...m], \qquad (6)$$

where *k* indicates the coefficient, and *m* is the order of the LP filter. The new $\tilde{\alpha}_k$ coefficients are mapped to $\tilde{\sigma}_k$ coefficients in the same manner as mentioned before [11]. We let *a* be 0.98, which results in a exponential decay of a^k. The smoothened filters (both large and small) were applied to the guitar samples. Moreover, each sound example was normalized in respect to their energy, with the inverse of the squared sum of all samples contained in one sound example. The duration of the guitar playing samples was two seconds, and the white noise and the impulse train, with three impulses, lasted a second each. In overall, there were twelve sample pairs to be matched.

5.2. Listening Test Results

Let the frequencies of the lowest body modes be hypothesized as guidelines to which direction warping should occur, in order to change the perceived size properly. Under these circumstances, when the observed size of the soundbox should be altered, a decrease in the warping coefficient, λ, should correspondingly resolve in an identifiably smaller sized instrument. In the same manner, an increment in λ should cause an enlargement in the apparent size of the guitar body. Figure 6 illustrates the listening test results, where the x-axis indicates each test case and the y-axis the values of λ. It shows that the direction of the adjusted warping parameter is in some degree consistent.

The median values for λ, from all the listening test results, are 0.713 and 0.742 for decrement and increment of the distinguished size of the guitar body, respectively. Hence, when comparing these values with the initial λ value, 0.73, the shift direction of the warping parameter is in line with the assumption of Lansky and Steiglitz. The inter-quartile range, IQR, expresses the range of half of the data. The IQR values, from all data, for shrinking and enlargement of the observed body size are 0.032 and 0.049, respectively.

From informal feedback from the subjects it can be concluded that reducing the perceived guitar body-size was easier and the

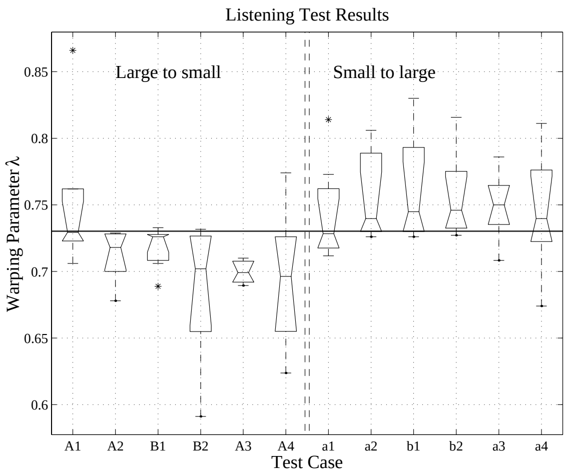

Figure 6: *Listening test results as a box-plot.*

resulting timbre was more pleasing to the testee than in the case of enlarging. In addition, the perceived size is more identifiable in the reduction case than in the enlargement one. This is due to a grown uncertainty of the perceived size when the value of λ is increased. This can be interpreted as a consequence of the shift of the higher resonances to lower frequencies. Due to the shift, the lowest body modes are also shifted and emphasize even lower frequencies, and therefore strengthen the perception of a larger instrument. At the same time, the strong resonances at higher frequencies also get shifted downwards on a nonlinear frequency scale and apparently increase the feel of a distinguishably small sized guitar. It can be argued on the strength of the listening test results and the informal feedback that the lowest body modes are more significant when identifying the size of a guitar body model than higher modes. Therefore the ambiguity of the perceived size is also dependent on which frequency range the listener concentrates on or gives more weight to. In overall, best results were obtained when the purpose was to shrink the size of the body model.

5.2.1. A More Detailed Look at the Listening Test Results

The vertical dashed lines in Fig. 6 separate the matching cases, where a large soundbox filter was adjusted to sound like a small one, from the opposite arrangement. These cases are displayed at the right and left hand side, respectively. The original λ value, 0.73, is indicated with a thick horizontal line. The alphanumeric indicators on the x-axis correspond to different excitations and filters in the following way: Letters A and B indicate the used filter, so that A is a straightforward WIIR and B is an A filter smoothed with Eq. 6, when $a = 0.98$. The numbers reveal the excitation signal, so that 1 and 2 correspond to picked and strummed guitar playing, respectively, and 3 stands for the impulse train, while 4 equals the white noise bursts. Furthermore, the cases where the large soundbox was matched with the small one are denoted with capital letters, and the small to large matching cases are displayed in lower case. In Figure 6, each box has lines at the upper (75%) and lower (25%) quartile values. Moreover, the median value is indicated between these values with a line and as a narrower part of each box. The whiskers are lines extending from each end of the box to show the extent of the rest of the data. Outliers, displayed with the symbol '*', are data with values beyond the ends of the whiskers.

The most systematic results were obtained when the excitation signal was an impulse train (see cases A3 and a3), even more so in case A3. All IQR values in the smoothed cases (B and b) are on the expected side of the initial λ value. Hence, the smoothing of the magnitude response can be considered as clarifying the perception of the size of the soundbox. Since test cases A1 and a1 were the first ones to be matched in the listening test it might be this that explains why these results behave poorly, rather than the lack of smoothing.

5.2.2. Spectral Interpretation of the Results

Figure 7 shows how the resonance structures have matched when the median λ values are observed together with original λ values. Panes (a) and (b) in Fig. 7 represent the magnitude responses in the shrinking case while screens (c) and (d) display them in the enlargement case. The so called target response is the topmost magnitude response in all the panes. The initial body filter is depicted as the bottom response and the warped (median valued) in the middle. Moreover, Figs. 7 (b) and (d) zoom to lower frequencies.

By examining Figures 7 and 3 one can see that the large and small sized body filters differ from each other. The most notable similar features are the three lowest resonances (80-500 Hz). The higher modes behave more irregularly. By viewing Fig. 7 it can be noted: the two lowest resonances move to the assumed direction, whereas the next strong resonances match better before warping. Moreover, the body-models differ from each other more than in the frequencies of their resonances: magnitudes, bandwidths, and the number of resonances vary. Therefore, it should be pointed out that by simply warping (change λ) a body filter does not result exactly in another filter.

5.3. Discussion

The wide deviation in the results obtained with white noise as an excitation signal (see cases A4 and a4 in Fig. 6) enforced the assumption that the weight given, unconsciously or by choice, to a certain frequency range can alter the final result or even end up in confusing ambiguity. The informal feedback also supports this assumption. The results behave better when guitar playing and impulses were used as excitation signals. This can again be understood through the effect of the excitation signal. With impulse excitation the sharpest resonances decay the slowest and effect the timbre the most. In our case the lowest body modes are the sharpest ones. The spectra of vibrating guitar strings are concentrated more on the lower frequencies. This could explain partly the behavior of the listening test results.

The number of resonances is not changed by warping even if the density of modes in a larger closed space is higher. Other modified methods with more control parameters might be developed that change the number of resonances depending on the direction of warping.

6. CONCLUSIONS

The idea of variable digital FIR-type filters was already introduced in [5] and for warped recursive filters it was applied (off line) in [8]. In most trivial warped FIR form the presented technique is also analogous to the traditional digital phaser effect [20]. However, so far this had not been done in real time for high-order all-pole models of an instrument body. Partly this is because the implementation of warped recursive filters is somewhat troublesome

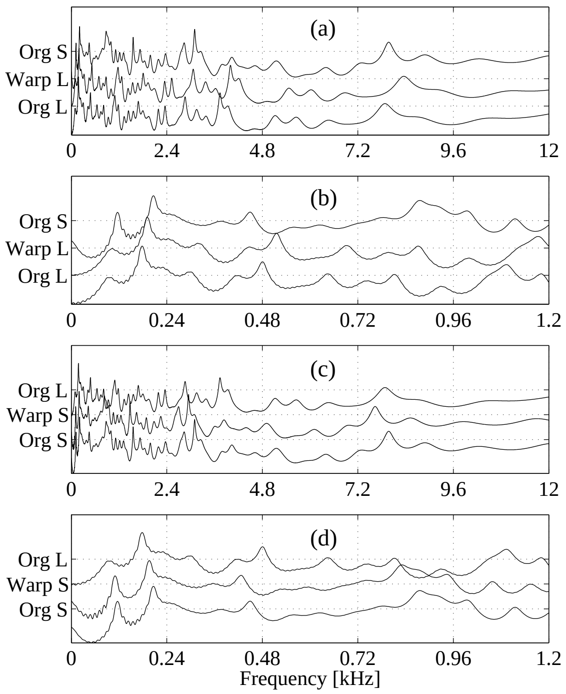

Figure 7: *Magnitude responses before and after matching the perceived sizes of the guitar bodies. Reducing the observed size: panes (a) and (b), and enlarging: panes (c) and (d).*

due to lag-free loops in the structure. In [12], a number of alternative techniques were introduced which finally makes real-time implementation of the presented continuously time-varying modulation approach plausible. Since the methods discussed in [17] and [18] produce filters that bring about the missing soundbox resonances so well, the techniques presented in this paper enable to alter the perceived size of the modeled soundbox. However, the change in the observed guitar size is not straightforward as the listening test results imply. Real-time alteration of the warping parameter also enables to create interesting sound effects that can be used for both the acoustic guitar and the electric guitar.

Sound examples related to this study can be found at www. acoustics.hut.fi/demo/dafx2000-bodymod/

7. ACKNOWLEDGEMENTS

Henri Penttinen has been working in the "Sound Source Modeling" project financed by the Academy of Finland. The work of Aki Härmä has been funded by the GETA graduate school.

8. REFERENCES

[1] M. Mathews and J. Kohut, "Electronic simulation of violin resonances," *Journal of the Acoustical Society of America*, vol. 53, no. 6, pp. 1620–1626, 1973.

[2] J. O. Smith, *Techniques for Digital Filter Design and System Identification with Application to the Violin*, Ph.D. thesis, Stanford University, 1983.

[3] M. Karjalainen and J. O. Smith, "Body modeling techniques for string instrument synthesis," in *Proc. Int. Computer Music Conf*, Hong Kong, 1996, pp. 232–239.

[4] M. Karjalainen, "Warped filter design for the body modeling and sound synthesis of string instruments," in *Proc. Nordic Acoustical Meeting (NAM '96)*, Helsinki, Finland, 1996, pp. 445–453.

[5] W. Schüssler and W. Winkelnkemper, "Variable digital filters," *Arch. Elek. Übertragung*, vol. 24, pp. 524–525, 1970, Reprinted in [21].

[6] H. W. Strube, "Linear prediction on a warped frequency scale," *Journal of the Acoustical Society of America*, vol. 68, no. 4, pp. 1071–1076, 1980.

[7] A. Härmä, M. Karjalainen, L. Savioja, V. Välimäki, U. K. Laine, and J. Huopaniemi, "Frequency-warped signal processing for audio applications," *J. Audio Eng. Soc.*, November 2000.

[8] P. Lansky and K. Steiglitz, "Synthesis of timbral families by warped linear prediction," *Computer Music Journal*, vol. 5, pp. 531–535, 1981.

[9] K. Steiglitz, "A note on variable recursive digital filters," *IEEE Trans. Acoust. Speech and Signal Proc.*, vol. 28, pp. 111–112, February 1980.

[10] M. Karjalainen, E. Piirilä, and A. Järvinen, "Loudspeaker response equalization using warped digital filters," in *Proc. of the IEEE Nordic Signal Proc. Symposium, NORSIG 96*, Espoo, Finland, September 1996, pp. 367–370.

[11] M. Karjalainen, A. Härmä, and U. K. Laine, "Realizable warped IIR filters and their properties," in *IEEE ICASSP'1997*, Munich, April 1997, vol. 3, pp. 2205–2209.

[12] A. Härmä, "Implementation of frequency-warped recursive filters," *Signal Processing*, vol. 80, no. 3, pp. 543–548, February 2000.

[13] E. Zwicker and H. Fastl, *Psychoacoustics— Facts and Models*, Springer, Berlin, 1990.

[14] J. O. Smith and J. S. Abel, "Bark and ERB Bilinear Transforms," *IEEE Trans. Speech and Audio Processing*, vol. 7, no. 6, pp. 697–708, 1999 Nov.

[15] C. Asavathiratham, P.E. Beckmann, and A.V. Oppenheim, "Frequency warping in the design and implementation of fixed-point audio equalizers," in *WASPAA'99*, New Platz, New York, October 1999, pp. 55–59.

[16] A. Härmä and M. Karjalainen, *Warp - Warping Toolbox for Matlab*, http://www.acoustics.hut.fi/software/warp/.

[17] M. Karjalainen, V. Välimäki, H. Räisänen, and H. Saastamoinen, "DSP equalization of electret film pickup for the acoustic guitar," in *106th AES Convention*, Munich, Germany, 1999.

[18] M. Karjalainen, H. Penttinen, and V. Välimäki, "Acoustic sound from the electric guitar using DSP techniques," in *IEEE ICASSP'2000*, Munich, Germany, 2000, pp. 773–776.

[19] T. Jungmann, "Theoretical and practical studies on the behaviour of electric guitar pickups," M.Sc. thesis, Helsinki University of Technology, 1994.

[20] J. O. Smith, "An allpass approach to digital phaseing and flanging," in *, Report STAN-M-21*, Stanford, CA, 1982.

[21] L. Rabiner and C. M. Rader, Eds., *Digital Signal Processing*, Selected Reprint Series. IEEE Press, New York, 1972.